and the acceleration (a) by

Elementary
Fluid Mechanics

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compressible and incompressible fluid motion.

18. The Energy Equation. Further information on flow in a streamtube can be gained by application of the principle of conservation of energy. This is merely a complete accounting of the various energy changes within a portion of a flow system and presents no difficulties once these energies have been identified. Considering a reasonably (but not completely[1]) general situation (Fig. 31), the possibility of heat energy, E_H, and mechanical energy, E_M, being added or extracted from the flow must be allowed; such energies will be taken to be the amount of energy added or extracted per pound (weight) of fluid. It follows from this that such energies must have dimensions of foot pounds per pound (ft-lb/lb) or the dimensional equivalent, ft.

The separate energies possessed by the flowing fluid are those associated with temperature, pressure, velocity, and height above datum. The internal energy, I, per pound of fluid is the energy associated with the kinetic energy of the molecules and the forces between them. The velocity, or kinetic, energy of one pound of fluid

[1] Chemical, electrical, and atomic energies are excluded from this analysis.

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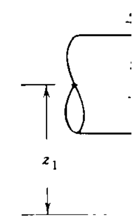

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Therefore

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may be written down directly from considerations of mechanics; the general expression for the kinetic energy of translation is $\frac{1}{2}MV^2$; for one pound of fluid however $M = 1/g$ so the kinetic energy becomes $V^2/2g$ ft-lb/lb. Again mechanics shows that the potential energy of a weight W at a vertical distance z above datum is (relative to the datum) Wz ft-lb; if the weight considered is one pound, the potential energy of the fluid is simply z ft-lb/lb. The energy

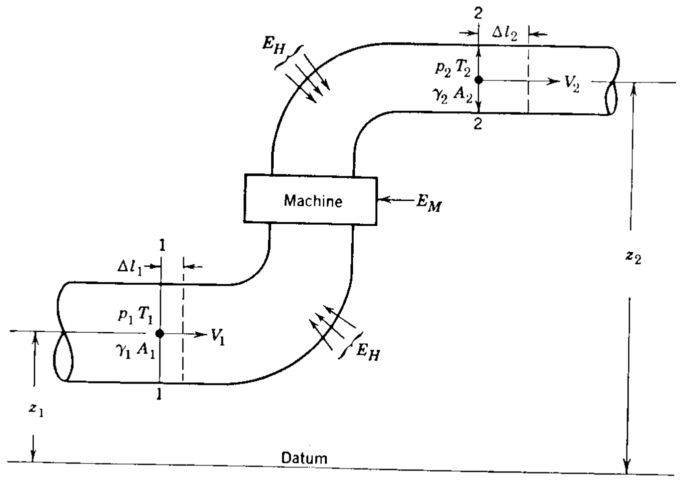

FIG. 31

contained in each pound of fluid may therefore be expressed as $\left(I + \frac{V^2}{2g} + z\right)$ ft-lb/lb. Restricting the discussion to steady flow and considering the continuity principle, each pound of fluid entering the boundary 1221 of Fig. 31 through section 1 must displace a pound of fluid which will move out of the boundary across section 2. Therefore

$$\begin{bmatrix}\text{Energy carried in by} \\ \text{each pound of fluid}\end{bmatrix} = I_1 + \frac{{V_1}^2}{2g} + z_1$$

$$\begin{bmatrix}\text{Energy carried out by} \\ \text{each pound of fluid}\end{bmatrix} = I_2 + \frac{{V_2}^2}{2g} + z_2$$

Consider now the work done on the fluid within the boundary by a weight of fluid entering the boundary and that done by the fluid within the boundary on the same weight of fluid leaving. Suppose that in a certain time a weight of fluid $(A_1\gamma_1\,\Delta l_1)$ moves into the boundary across section 1 and that in this time a weight of fluid ($A_2\gamma_2\,\Delta l_2$) leaves the boundary across section 2. The work associated with these displacements can be computed from the products of forces (p_1A_1 and p_2A_2) and displacements (Δl_1 and Δl_2) which give $p_1A_1\,\Delta l_1$ and $p_2A_2\,\Delta l_2$, respectively. For inclusion in the energy equation these work terms must be written per pound of fluid flowing; this may be done by dividing by the number of pounds of fluid ($A_1\gamma_1\,\Delta l_1$ and $A_2\gamma_2\,\Delta l_2$) doing the work, giving p_1/γ_1 and p_2/γ_2, respectively. From this development it is evident that a pound of fluid entering the boundary at 1 does work p_1/γ_1 on the fluid within the boundary and that the fluid within the boundary does work p_2/γ_2 on a pound of fluid leaving at section 2. From this it follows that the net work done on the fluid within the boundary is ($p_1/\gamma_1 - p_2/\gamma_2$), which may be written as ($p_1v_1 - p_2v_2$), the product pv being generally known as the "flow work."

The energy equation may now be written (Fig. 31) as

$$\begin{bmatrix}\text{Energy entering}\\ \text{the boundary}\\ 1221\end{bmatrix} + \begin{bmatrix}\text{Net flow work done}\\ \text{on fluid within the}\\ \text{boundary } 1221\end{bmatrix} = \begin{bmatrix}\text{Energy leaving}\\ \text{the boundary}\\ 1221\end{bmatrix}$$

or

$$\left[I_1 + \frac{V_1^2}{2g} + z_1 + E_H + E_M\right] + \left[p_1v_1 - p_2v_2\right] = \left[I_2 + \frac{V_2^2}{2g} + z_2\right] \tag{33}$$

or, collecting terms of the same subscript and replacing pv with p/γ,

$$I_1 + \frac{p_1}{\gamma_1} + \frac{V_1^2}{2g} + z_1 + E_H + E_M = I_2 + \frac{p_2}{\gamma_2} + \frac{V_2^2}{2g} + z_2 \tag{33}$$

With the energy equation written in the pattern of equation 33, the flow work terms p/γ become associated with the energy terms I, $V^2/2g$, and z; and because of this p/γ is generally (and conveniently) treated as the "pressure energy" of the flow. Although the foregoing development has shown that p/γ is not strictly a pressure energy its use leads to no practical difficulty in fluid flow problems.[2]

Assuming an increase in all of the separate energies from section 1 to section 2, equation 33 may be written in differential form,

$$dI + d(p/\gamma) + d(V^2/2g) + dz = dE_H + dE_M \tag{34}$$

[2] In nonflow systems the flow work term does not appear, so its equivalent pressure energy has no meaning in such problems.

$\Delta I + \Delta PV + \Delta \frac{V^2}{2} + \Delta gz = W + Q$

$W = Q - \Delta h - \Delta \frac{V^2}{2} - \Delta gz$

which is convenient for comparison with the Euler equation. All of the terms of equation 34 can be easily visualized as applying to a streamtube element of differential size, with the possible exception of dE_M which implies the existence of an infinitesimal machine and leads to a contradiction since in practice mechanical energy is added in large "lumps" rather than in differential quantities. For this reason it is not inconsistent to omit dE_M from equation 34 but to include E_M in equation 33. Omitting dE_M, equation 34 becomes

$$dI + d(p/\gamma) + d(V^2/2g) + dz = dE_H \tag{34a}$$

19. Comparison of the Energy and Euler Equations. Comparison of equations 32 and 34a leads directly to some useful conclusions. Restating these equations,

$$v\,dp + d(V^2/2g) + dz + (\tau/\gamma R)\,dl = 0 \tag{32}$$

$$dI + p\,dv + v\,dp + d(V^2/2g) + dz - dE_H = 0 \tag{34a}$$

it is at once evident that they have three common terms.

For the flow of liquids, where change of density and specific volume may be ignored, dv will be zero causing $p\,dv$ to vanish and leading to the conclusion that

$$(\tau/\gamma R)\,dl = dI - dE_H$$

Later it will be shown that $(\tau/\gamma R)\,dl$ is known as the "head loss" or "energy dissipated by friction"; the equation offers proof that head loss is not a loss of total energy but rather a conversion of energy into heat, part of which leaves the fluid and the remainder serving to increase its internal energy. This is the practical case of incompressible flow as it appears in many engineering applications; here head loss is a permissible and useful concept, because heat energy leaving the flow and energy converted into internal energy are seldom recoverable and are in effect lost from the useful total of pressure, velocity, and potential energies. For compressible fluid motion this is not generally true, since the useful total of energies will include the internal energy.

Another fruitful comparison between the energy and Euler equations may be made by considering the condition of frictionless flow. Here $\tau = 0$ and the Euler equation reduces to

$$v\,dp + d(V^2/2g) + dz = 0$$