Angular Momentum and Circulation

Angular Momentum (M) and Circulation (Γ) are conserved in frictionless converging flow. Angular momentum is the product radius (r) and tangential velocity (v). Circulation is the integral of tangential velocity and path length. For a uniform circular path, circulation equals angular momentum multiplied by 2π.

$$v_a \; r_a = v_s \; r_s$$

The absolute tangential velocity (v_s) of air stationary relative to the earth surface is:

$$v_s = \frac{2\pi \; r_s \; \sin\phi}{N}$$

$$r_s = \sqrt{\frac{r_a \; v_a \; N}{2\,\pi \sin(\phi)}} = \sqrt{\frac{r_a \; v_a \; N}{\pi}}$$

where v_a and r_a are the velocity and radius at the radius of maximum velocity; where v_s is the absolute tangential velocity of the earth's surface at a distance r_s from the axis of rotation, where N is the number of seconds per day (= 86400), and ϕ is the latitude. The latitude will be taken to be 30° so that sin (ϕ) is 0.5.

Table 1 shows the effect of circulation of annulus radius (r_a) for convective vortices with peak tangential velocities (v_a) of 50 m/s. The radius (r_s) in Table 1 is the radius at which air stationary relative to the earth's surface would have the same absolute circulation as the air at the radius of peak velocity.

Air stationary relative to the earth's surface at a radius of 262 km has an absolute tangential velocity of 9.5 m/s relative to the vortex axis. For the 50 km radius hurricane at the bottom of the table, the air has to converge from a distance of 262 km to produce an eyewall tangential velocity of 50 m/s. For the 1 m radius dust devil at the top of the table, the air has to converge from a distance of 1.2 km to produce a peak tangential velocity of 50 m/s. Producing a tangential velocity of 50 m/s requires that the air converges by a factor of 5.24 in the case of the hurricane and by a factor of 1171 in the case of the dust devil.

Table 1 – Distance (r_s) from which air stationary relative to the earth's surface has to converge to have a peak tangential velocity (v_a) of 50 m/s.

Vortex peak tangential velocity	Vortex radius at peak velocity	Angular Momentum	Source air radius	Source air absolute tangential velocity	Radius & velocity Ratios
v_a (m/s)	r_a (m)	M (m^2/s)	r_s (m)	v_s (m/s)	r_s/r_a
50	1	50	1,170	0.04	1173
50	10	500	3,700	0.13	371
50	100	5,000	11,700	0.43	117
50	1,000	50,000	37,000	1.35	37.1
50	10,000	500,000	117,000	4.26	11.7
50	20,000	1,000,000	166,000	6.03	8.29
50	50,000	2,500,000	262,000	9.53	5.24

Surface winds, which typically have velocities of less than 3 m/s, have little effect on absolute circulation when the converging air comes from a location where the absolute tangential velocity of the earth's surface is more than 4 m/s (r_s > 10 km). Local wind can have a significant effect on circulation when the converging air comes from a location where the absolute tangential velocity of the earth's surface in under 1 m/s (r_s < 1 km).

The diameter of a vortex increases with circulation, and the circulation of the converging air increases with its initial distance from the vortex axis. The circulation of a hurricane results from convergence of air initially rotating at close to the rotation velocity of the earth's surface. The circulation of dust devils results form convergence in air masses swirling relative to the earth's surface. The initial swirling action could be the result of the vortex forming within a random eddy or near the boundary between two air streams. Hurricanes always have cyclonic circulation. Dust devils can have cyclonic or anti-cyclonic circulation. Intermediate size tornadoes predominantly have cyclonic circulation but occasionally have anti-cyclonic circulation. While the rotation of the earth's surface plays a major role in large vortices like hurricanes, the earth's rotation has little effect on small vortices like dust devils.

The high tangential velocity of convective vortices is always the result of convergence in a relatively thin layer of air close to the earth's surface: in the case of hurricane, of air initially rotating at the same speed as the earth's surface; in the case of dust devils, of randomly rotating air masses. Convergence does not necessarily produce tight convective vortices. Convergence in a thunder storm can produce rotation in large air masses such as rotating super cells. Super cell rotation does not cause tornado; like tornadoes super cell rotation is caused by convergence. Convergence in a thin layer produces the rotation in both super cells and tornadoes. Without convergence the rotation of a large air mass would be dampened by friction.

Friction between the air converging towards the base of the vortex and earth's surface reduces the circulation of the converging air. As a result the converging air would have to come from a radius somewhat higher than 262 km to have a tangential velocity of 50 m/s at a radius of 50 km. Hurricane diameter increase during their intensification period indicating that the distance from which the air converge increases as the hurricane intensifies.

LMM 2010-01-04