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Total energy equation method for calculating hurricane intensity

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With 2 Figures

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Summary

Surface pressure reduction in hurricanes is calculated by applying the total energy equation (TEE) to ideal isentropic upflow in a vertical tube. The pressure reduction at the base of the tube, called the *intensity*, is calculated for three upflow processes: *reversible* upflow of air approaching equilibrium with the sea at the sea level pressure outside the tube; *irreversible* upflow of air approaching equilibrium with the sea at the sea-level pressure outside the tube; and upflow of air approaching equilibrium with the sea at the reduced surface pressure inside the tube. The sensitivity of *intensity* to the type of upflow process and to the sea surface temperature is investigated. Intensities calculated with the TEE are shown to be consistent with observations and to be close to intensities calculated with more complex methods. The TEE method is simple and can help understand the basic mechanism responsible for surface pressure reduction and for energy production. The method is used to show that approximately 20% of the heat taken from the sea during a hurricane is converted to mechanical energy.

1. Introduction

Holland (1997) calculated tropical cyclone surface pressure by integrating the hydrostatic equation downward from the level of neutral buoyancy. He developed an iterative method for calculating minimum hurricane surface pressure which he called *Maximum Potential Intensity* (MPI). This paper shows that MPI's agreeing with Holland's can be obtained by simply applying the total energy equation to upward flow in a vertical tube. Pressure reduction at the surface, called *intensity*, will be used as the measure of *Hurricane Intensity (HI)*.

In tropical-oceanic areas, the air near the surface approaches equilibrium with the sea; in light to moderate wind, the surface air temperature (SAT) is typically 1 °C lower than the sea surface temperature (SST) and the surface air has a relative humidity (U) of approximately 80%. High winds in hurricanes reduce the approach to equilibrium because the air comes in more intimate contact with the water. Holland (1997) used eyewall air temperature 1 K lower than the SST and eyewall relative humidity of 90%; the values used by Holland will be used initially to facilitate comparison. Cione and Black (1998) used observational data to show that as the eyewall of the hurricane is approached: the air-sea contrast (ACS = SST − SAT) increase from 1 to 3 °C, and the surface air relative humidity (SAU) increases from 80 to 90%. It is common to assume that a gas in close contact with a liquid, such as the air coming out of a cooling tower, approaches equilibrium with the water. The relative humidity of surface air at hurricane eyewall is difficult to measure, but can be inferred from minimum hurricane cloud height which is around 300 m, Holland (1997). The lifting condensation level of air with 90% relative humidity is approximately 300 m; therefore the air rising in the eyewall must have a relative humidity close to 90%.

Holland (1997) pointed out that the release of all latent-heat energy of air at normal surface pressure and humidity can only reduce eyewall pressure by 4 kPa and that intense tropical cyclones must obtain additional energy from air-sea interaction at eyewall pressure. Pressure has a large effect on mixing ratio. The mixing ratio (r) of air at 30 °C and 90% relative humidity is 24.5 g kg^{-1} at 101 kPa and 29.4 g kg^{-1} at 85 kPa. Increasing mixing ratio increases enthalpy and hurricane intensity.

2. Thermodynamic basis

The three steady-state ideal thermodynamic flow processes shown in Fig. 1 will be analyzed. In each case, surface air is raised isentropically in an insulated vertical tube and the flow is restricted at bottom of the tube, see Michaud (2000). Holland's (1997) calculations were based on the rising air having a constant equivalent potential temperature (θ_e); the total energy equation method is based on the rising air having a constant entropy (s). The two methods are equivalent since a constant equivalent potential temperature process is an isentropic process. The hydrostatic method of calculating surface pressure reduction implies that the rising air is separated from the environment by a partition such as the wall of the tube in Fig. 1.

Reversible case A corresponds to *true-adiabatic* expansion. Reversibility requires that kinetic energy be removed from the system before dissipating. The turbine in Fig. 1a captures the kinetic energy of the jet (w_x) coming out of the opening in the base of the tube. In irreversible case B the kinetic energy of the jet is not captured and is allowed to revert to heat; process 1–2 changes from a *constant entropy* process, $s_2 = s_1$, to a *constant enthalpy* process, $h_2 = h_1$ which is also a constant temperature process, $T_2 = T_1$. In *reversible* case A and *irreversible* case B, the air has a relative humidity of 80% and a temperature 1 °C lower than SST at P_1 before the start of the expansion. In *air-sea interaction* case C, the air has a relative humidity of 90% and a temperature 1 °C lower than SST at reduced pressure P_2 after the initial expansion.

Work in a continuous flow process is given by the total energy equation:

$$w = q - \Delta h - \Delta gz - \Delta \frac{v^2}{2}, \quad (1)$$

where w is the work given up by the flowing fluid, q is the heat received by the flowing fluid, h is the enthalpy of the air including the enthalpy of its water content, g is the acceleration of gravity, z is the height, and v is the velocity. The deltas (Δ) are taken between the inlet and outlet conditions of each process. Equation (1) is applicable to individual processes 1–2 and 2–3 and to the overall process 1–3. For processes without heat transfer and negligible velocities, $q = 0$ and v approaches zero; (1) reduces to:

$$w = -\Delta h - \Delta gz = -\Delta\mu, \quad (2)$$

where μ is the *static-energy* of the air including its water content, where $\mu = h + gz$.

The TEE method is based on the realization that the work ($w_{23} = 0$) during constant entropy upflow process 2–3 approaches zero. The pressure drop due to friction in the upflow process is small compared to the hydrostatic surface pressure reduction. The pressure drop due to friction in a horizontal tube 1 km in diameter by 15 km long at an air velocity of 20 m s^{-1} is less than 0.01 kPa. The pressure at the base of the tube, P_2, is there-

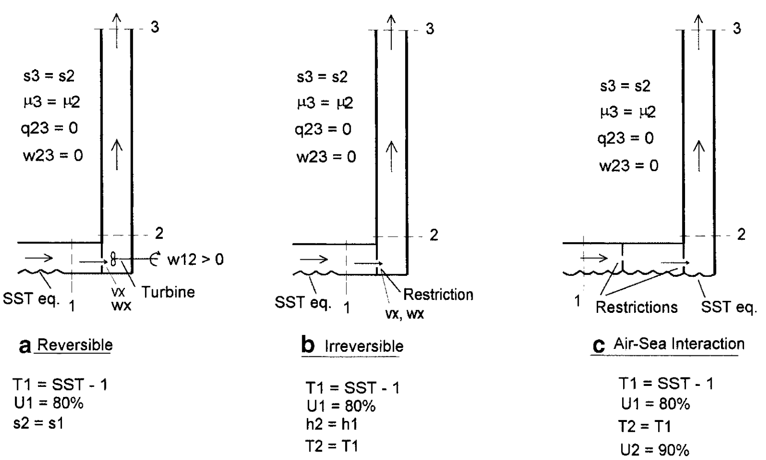

Fig. 1. Ideal upflow process in a vertical tube. **a** Reversible case, air approaching equilibrium with the sea at ambient pressure; **b** Irreversible case, air approaching equilibrium with the sea at ambient pressure; **c** Air-sea interaction case, air approaching equilibrium with the sea at the reduced pressure inside the base of the tube

Table 1. MPI and Base pressure reduction when air is raised from sea level to the *20 kPa* level.

Air Properties	Reversible case A	Irreversible case B	Air-sea case C
SST (C)	28	28	28
P_1 (kPa)	101.0	101.0	101.0
SAT $= T_1$ (C)	27.0	27.0	27.0
T_1 (K)	300.15	300.15	300.15
U_1 (%)	**80**	**80**	**80**
r_1 ($\mathrm{g\,kg^{-1}}$)	18.18	18.18	18.18
$\mu_1 = h_1$ ($\mathrm{J\,kg^{-1}}$)	73485	73485	73485
s_1 ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	256.7	256.7	256.7
MPI $= P_2$ (kPa)	**99.06**	**98.48**	**96.29**
T_2 (K)	**298.50**	**300.15**	**300.15**
r_2 ($\mathrm{g\,kg^{-1}}$)	18.18	18.18	21.49
U_2 (%)	86.7	77.9	**90**
$\mu_2 = h_2$ ($\mathrm{J\,kg^{-1}}$)	**71775**	**73485**	**81931**
s_2 ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	**256.7**	**264.4**	**299.3**
P_3 (kPa)	20.0	20.0	20.0
T_3 (K)	228.50	230.02	237.94
r_3 ($\mathrm{g\,kg^{-1}}$)	18.18	18.18	21.49
z_3 (m)	12400	12400	12400
h_3 ($\mathrm{J\,kg^{-1}}$)	− 51954	− 50243	− 42201
$\mu_3 = h_3 + gz_3(1 + r)$	71775	73485	81931
s_3 ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	256.7	264.2	299.3
Work ($\mathrm{J\,kg^{-1}}$)			
$\Delta h_{23} = h_2 - h_3$	123730	123730	124130
$\Delta gz_{23} = gz(1 + r)$	123730	123730	124130
w_{23}	0	0	0
w_x	1710	2232	4230
Velocity ($\mathrm{m\,s^{-1}}$)			
v_x	58.5	66.8	92.0
Base Pressure Reduction			
$HI = \Delta P_{12}$ (kPa)	1.94	2.52	4.71

fore the pressure for which the work during process 2–3 approaches zero, the pressure for which the change in the static-energy of the rising air is zero. The same procedure is used to calculate base pressure in the three cases. A solver is used to calculate the work during upflow process 2–3 (w_{23}) for two P_2 guesses; linear interpolation is then used to calculate the pressure P_2 for which w_{23} is zero. The bottom pressure P_2 was subsequently verified independently by downward integration of the hydrostatic equation from the top of the tube to the surface.

Table 1 shows the results for air raised form the surface to the 20 kPa level under typical tropical-oceanic conditions. The geo-potential height of the 20 kPa surface is taken as 12400 m, which is typical for oceanic-tropical areas. As will be shown later, the intensity is higher if the air is raised to its level of neutral buoyancy. The level of neutral buoyancy in tropical-oceanic areas is usually above the 20 kPa level. The equations used to calculate the thermodynamic properties of the air are given in Appendix A of Michaud (1995)[1].

3. Results

3.1 Reversible case A

The conditions of the surface air in Table 1 are: $P_1 = 101\,\mathrm{kPa}, T_1 = 27\,°\mathrm{C}, U_1 = 80\%$. The MPI is 99.06 kPa, and the *intensity* is 1.94 kPa. The entropy (s) of the air is $256.7\,\mathrm{J\,kg^{-1}}$ in the three states. Intensity is very sensitive to surface temperature and humidity. Increasing SAT by 1 °C at

[1]The Mathcad program used to calculate Tables 1 and 2 and to produce Fig. 2 is on web site: http://www3.sympatico.ca/louis.michaud/Calculation/AT_MCD.htm# Hurricane MPI

constant mixing ratio (r) increases intensity by 0.27 kPa. Increasing SAU by 1% at constant SAT increases intensity by 0.11 kPa. Increasing SAT by 1 °C at constant SAU increases intensity by 0.90 kPa because both the temperature and the mixing ratio increase. Decreasing the 20 kPa geo-potential height by 100 m by decreasing the average temperature of the environment by ≈ 2 K increases intensity by 1.11 kPa. A small change in air temperature has a large effect on intensity. Decreasing the SAT by 2 °C at constant relative humidity would decrease the intensity to close to zero. Case A correspond to customary true-adiabatic lifting, which is an idealization since in nature work is not removed from the system; the work ($w_{12} = 1710\,\mathrm{J\,kg^{-1}}$) corresponds to CAPE.

3.2 Irreversible case B

In irreversible case B, the MPI is 98.48 kPa, and the *intensity* is 2.52 kPa. The entropy of the air increases from 256.7 to 264.4 $\mathrm{J\,K^{-1}\,kg^{-1}}$ as the kinetic energy is dissipated in irreversible process 1–2. The sensitivity is higher than in reversible Case A. Increasing SAT by 1 °C at constant mixing ratio increases intensity by 0.35 kPa. Increasing SAU by 1% at constant SAT increases intensity by 0.16 kPa. Increasing SAT by 1 °C at constant SAU increases intensity by 1.13 kPa.

Irreversibility increases the intensity by 30%. The work dissipated $w_x = 2232\,\mathrm{J\,kg^{-1}}$ is 30% higher than the work in the reversible case; the velocity of the air at the restriction outlet increases by 14%. Bister and Emanuel (1998) used numerical models to show that dissipative heating can increase the maximum wind speed in hurricanes by 20%. They argued that the bulk of the dissipative heating occurs in the atmospheric boundary layer near the radius of maximum wind. Constant enthalpy process 1–2 does not have to be a single restriction, adiabatic dissipative processes are usually constant enthalpy processes. There could be several restrictions in the conduit in which case the kinetic energy at the outlet of each restriction would only be part of the total kinetic energy. The kinetic energy at the outlet of the single restriction is a good indication of the total mechanical energy dissipated, but kinetic energy is not usually a good indication of the energy dissipated in a constant enthalpy process because the mechanical can be dissipated in stages. The kinetic energy at the outlet of the single restriction is equal to the reduction in enthalpy during isentropic expansion from P_1 to P_2. State x can be considered as an intermediate state between states 1 and 2 where the kinetic energy has not yet dissipated.

3.3 Air-sea interaction case C

The level of neutral buoyancy for air approaching equilibrium with water at the reduced surface pressure is closer to 10 kPa than to 20 kPa. The height of the 10 kPa geo-potential surface in tropical-oceanic areas is typically 16600 m. Table 1 shows the base pressure when the air is raised to the 20 kPa level, and Table 2 shows the base pressure when the air is raised to the 10 kPa level. For the 27 °C SAT and 90% relative humidity cases, the intensity is 4.71 kPa when the air is raised to the 20 kPa level and 7.78 kPa when the air is raised to the 10 kPa level.

Table 2 shows that *intensity* in the air-sea interaction case is extremely sensitive to surface temperature and humidity. Increasing SAT by 1 °C at constant relative humidity increases intensity by **2.9 kPa**. Increasing the relative humidity at constant surface air temperature by 1% increases *intensity* by **0.4 kPa**. Holland hydrostatic method calculates the pressure at the *eye*, the TEE method calculates the pressure at the *eyewall* where the upflow takes place; the intensity at the eye can be 20% to 30% higher than the intensity at the eyewall. Holland (1997) obtained the following eye-MPI sensitivities for a mean January Willis island sounding, see his Table 5: sensitivity of MPI to SST: $\mathbf{3.3\,kPa\,K^{-1}}$, sensitivity of MPI to SAU: $\mathbf{0.48\,kPa\,\%^{-1}}$.

Holland (1997) showed that the results of his hydrostatic method agree with an empirical relation between SST and MPI derived from observations by DeMaria and Kaplan (1994). For 90% SAU and SST of 27 and 29 °C, respectively: DeMaria and Kaplan's empirical relation gives eye-MPI of 94.6 and 91.4 kPa, for a sensitivity of $\mathbf{1.6\,kPa\,K^{-1}}$, see Willoughby and Black (1996), fig. 1; Holland's hydrostatic method gives eye-MPI's of 96.3 and 89.7 kPa, see Holland's table 5; the TEE method gives eyewall-MPI's of 96.4 and 90.3 kPa.

The fact that the minimum SST required for hurricane development is 26 °C is well recognized,

Table 2. MPI and Base Pressure Reduction with *air-sea interaction*, Case C, air raised from sea level to the *10 kPa* level. $T_1 = T_2 = SST - 1$

Air Properties	Base	$U = U_b + 5\%$	$T = T_b + 1\,\mathrm{K}$
SST (C)	**28.0**	**28.0**	**29.0**
P_1 (kPa)	101.0	101.0	101.0
SAT $= T_1$ (C)	**27.0**	**27**	**28.0**
T_1 (K)	300.15	300.15	301.15
U_1 (%)	80	80	80
r_1 (g kg^{-1})	18.18	18.18	19.31
$\mu_1 = h_1$ (J kg^{-1})	73485	73485	77422
s_1 (J K^{-1} kg^{-1})	256.7	256.7	269.9
MPI $= P_2$ (kPa)	**93.22**	**91.21**	**90.29**
T_2 (K)	**300.15**	**300.15**	**301.15**
U_2 (%)	**90.0**	**95.00**	**90.0**
r_2 (g kg^{-1})	22.22	24.00	24.41
$\mu_2 = h_2$ (J kg^{-1})	83807	88330	90446
s_2 (J K^{-1} kg^{-1})	315.2	336.9	346.9
P_3 (kPa)	10.0	10.0	10.0
T_3 (K)	201.69	206.48	208.59
r_3 (g kg^{-1})	22.22	24.00	24.41
z_3 (m)	16600	16600	16600
h_3 (J kg^{-1})	−82488	−78252	−76206
$\mu_3 = h_3 + gz_3(1 + r)$	83807	88330	90446
s_3 (J K^{-1} kg^{-1})	315.2	336.9	346.9
Work (J kg^{-1})			
$\Delta h_{23} = h_2 - h_3$	166295	166582	166652
$\Delta gz_{23} = gz(1 + r)$	166295	166582	166652
$w_{23} = \Delta\mu_{23} = \Delta h_{23} - \Delta gz_{23}$	0	0	0
w_x	7074	8997	9909
Δw_x	base	1923	2835
Velocity (m s^{-1})			
v_x	118.9	134.1	140.8
Δv_x (%)	base	12.8	18.4
Base Pressure Reduction (kPa)			
$HI = \Delta P_{12}$	7.78	9.79	10.71
ΔP_2	base	2.01	2.93
Sensitivity			
$\Delta P_2/\Delta T_2 = 2.93$ kPa/K versus 3.3 kPa/K in Holland (1997) Table 5			
$\Delta P_2/\Delta U_2 = 0.40$ kPa/% versus 0.48 kPa/% in Holland (1997) Table 5			
$\Delta w_x/\Delta T_2 = 2835$ J kg^{-1}/K			
$\Delta w_x/\Delta U_2 = 385$ J kg^{-1}/%			

Henderson-Sellers et al. (1998). Reducing the SAT to 25 °C reduces the intensity to close to zero. A SAT of 30 °C gives an eyewall MPI of 83 kPa sufficient to account for the lowest observed hurricane pressure. The higher intensity in the air-sea interaction case is mainly due to the fact that for the same temperature and relative humidity air holds more water vapor at lower pressure.

The double restriction shown in the air-sea interaction case of Fig. 1 represent the fact that the work can be dissipated in stages. Hurricane minimum pressures are in close agreement with Table 2; hurricane maximum velocities are lower than in Table 2 because the work is dissipated in stages as the air converges towards the eyewall. For the $P_3 = 20$ kPa case, the kinetic energy at the restriction outlet is, $w_x = 4230\,\mathrm{J\,kg^{-1}}$, 2.5 times the kinetic energy in the reversible process. For air raised to the 10 kPa level, the kinetic energy is $7074\,\mathrm{J\,kg^{-1}}$ for 27 °C SAT and $9909\,\mathrm{J\,kg^{-1}}$ for 28 °C SAT.

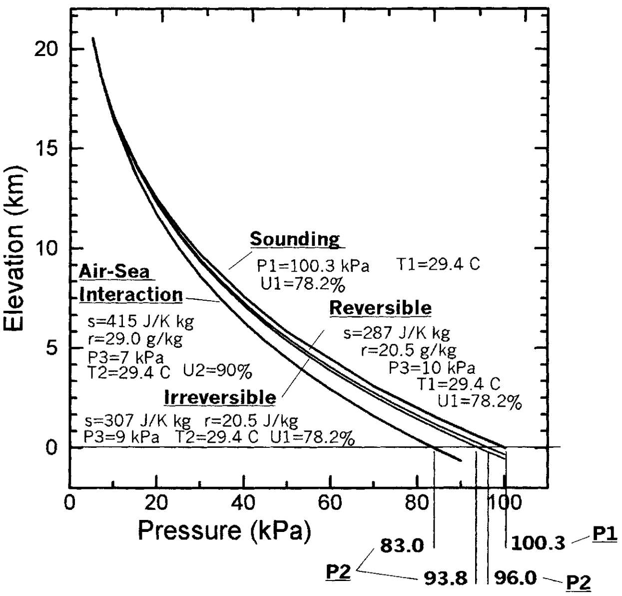

Fig. 2. Pressure versus elevation plot for the Willis island sounding of 0000Z, 17 January, 1999. The plot shows the properties of the rising air, the level of neutral buoyancy, and the base pressure reduction for the three upflow processes

Holland (1997) applied his method to a mean Willis Island January sounding. The total energy equation (TEE) method was applied to a specific sounding, the January 17, 1999, 0000Z Willis island sounding; fig. 2 shows the pressure inside the tube and the ambient pressure. The conditions of the rising air without air-sea interaction was taken as the base sounding conditions: $P = 100.3$ kPa, $T = 29.4$ °C, $U = 78.2\%$. The MPI is 93.8 kPa for the irreversible process; and 96 kPa for the reversible process. The base pressure reduction was calculated using the two guesses method assuming a tube top pressure of 10 kPa. The buoyancy of the lifted air at the tube top was then checked and the tube top height adjusted until the tube reached the level of neutral buoyancy. The level of neutral buoyancy is 10 kPa for the reversible case, 9 kPa for the irreversible case, and 7 kPa for the air-sea interaction case. The surface pressure in the air-sea interaction case is 83 kPa. Figure 3 of Henderson-Sellers et al. (1998) shows a MPI of 88 kPa for mean January Willis island soundings.

The intensity calculation are based on the condensed water freezing and not separating from the air, *true-adiabatic* expansion. Separating the water from the air, *pseudo-adiabatic* expansion, has little effect on *intensity*. More mechanical energy is produced in true-adiabatic expansion because more heat is transferred from the condensed water to the air, but the additional energy is used to lift water and does not appreciably increase intensity.

4. Discussion

4.1 Effect of reduction in SST and of SAC

Intensity depends mainly on the temperature and the humidity of the air at the base of the tube. The intensity in the first column of Table 2 applies whether the SST is 28 °C or 31 °C so long as the air at the base of the tube has a relative humidity of 90% and a temperature of 27 °C. Cione and Black (1998) showed from observational data that the relative humidity of the air at the eyewall is usually close to 90%, and that the SAC increases from approximately 1 °C to 3 °C as the eyewall is approached; the SAC increases with wind intensity. The DeMaria and Kaplan (1994) empirical relation was approximated with the following linear equation from fig. 1 of Willoughby and Black (1996).

$$MPI \approx 137.8 - 1.6\,SST \quad 26 < SST < 30. \qquad (3)$$

The results of the TEE method can be summarized with a similar linear equation:

$$MPI \approx 171.5 - 2.9\,SAT \quad 25 < SAT < 29. \qquad (4)$$

In order for (4) to give the same result as (3) the SAC must be 1.8 °C at SST of 30 °C and 0.5 °C at SST of 27 °C. The SAC's required to make the MPI agree are slightly lower than the Cione and Black (1998) SAC's, but they support their observations that SAC increase with SST.

The passage of a hurricane can reduce SST by 2 to 4 °C, Black and Shay (1998). Taking the SST reduction as 2 °C and the SAC as 2 °C, the temperature of the rising air could be 4 °C lower than the SST prior to the hurricane. The combined decrease of 3 °C in SAT would reduce intensity by 8.7 kPa. Schade and Emanuel (1999) and Chan et al. (2001) showed that SST reduction can reduce MPI by approximately 50%. The sensitivity of intensity to the pre-storm SST is probably between 1.0 and 2.0 kPa K^{-1}, because of the decrease in SST and because the SAC is higher than 1 °C near the eyewall of intense hurricanes. The sensitivity to pre-storm SST ranges from

$2.0\,\mathrm{kPa\,K^{-1}}$ for fast moving hurricanes with a deep warm layer to $1.0\,\mathrm{kPa\,K^{-1}}$ for slow moving hurricanes with shallow warm layer because fast moving hurricane have little effect on a deep warm layer.

Chan et al. (2001) developed a coupled atmosphere-model capable of simulating the main characteristics of tropical cyclones including the decrease in sea temperature. They obtained sensitivities to the SST prior to the storm of $1.6\,\mathrm{kPa\,K^{-1}}$ in their coupled model where the hurricane is allowed to reduce the SST. Their model shows that a hurricane can reduce the temperature of a surface layer 50 to 100 m thick by 3.6 °C. The heat flux required to cool a layer 60 m thick by 3.5 °C in one day is approximately $10000\,\mathrm{W\,m^{-2}}$. The heat transfer rate between the sea and the air at the eyewall can be extremely high. A heat flux of $10000\,\mathrm{W\,m^{-2}}$ is two orders of magnitude larger the normal latent heat flux from the tropical ocean surface. Shay et al. (1998) estimated the heat loss from the warm ocean layer of hurricane Opal at $15000\,\mathrm{W\,m^{-2}}$. Direct contact between spray and air can greatly increase heat transfer. The heat received per unit mass of air in the base case of Table 2 is $10000\,\mathrm{J\,kg^{-1}}$ $(h_2 - h_1)$. An upward air velocity of $1\,\mathrm{m\,s^{-1}}$ at $10000\,\mathrm{J\,kg^{-1}}$ would give an upward heat flux of $10000\,\mathrm{W\,m^{-2}}$. The high heat transfer could result from spray being cooled by evaporation, falling back into the sea, and then sinking through the warm surface layer. The minimum temperature to which water can be cooled in a cooling tower is the wet bulb temperature of the air. The wet bulb temperature of air with 90% relative humidity is approximately 1 °C lower than its dry bulb temperature. The temperature of the spray settling back on the sea could be 1 °C lower than the temperature of the spray taken from the sea resulting in a very high heat transfer rate. The fact that the energy of hurricanes is the result of heat transfer from sea to air is not universally recognized; the SST reduction is generally attributed entrainment of cold water from below, see Shay et al. (1998) and Shade (2000). Entrainment of cold water from below requires that dense cold water ascend through lower density warm water; surface wind is unlikely to be able to cause a significant amount of denser water to rise through less dense water at depths of 50 to 100 m. The reduction in the temperature of the warm ocean layer is more likely to be due to the sinking of cold water than to the rising of cold water.

4.2 Mechanical energy

Holland's hydrostatic method calculates surface pressure reduction, but does not attempt to calculate mechanical energy production. The TEE method implicitly recognizes that mechanical energy is produced and dissipated. Mechanical energy is produced when heat is carried upward because there is heat flow from a hot source, the sea; to a cold source, the upper troposphere, see Renno and Ingersoll (1996). The heat to work conversion efficiency is the Carnot efficiency (n) calculated using the SST as the hot source temperature (T_h) and the temperature of the upper troposphere as the cold source temperature (T_c), $(n = 1 - T_\mathrm{c}/T_\mathrm{h})$. Taking the hot source temperature as 300 K and the cold source as 230 K, the Carnot efficiency is 23%; of which about 5% is required to lift water leaving about 18% to produce motion and overcome frictional resistance to flow. The cold source temperature is somewhat less than the average temperature of the troposphere because subsidence warming is more effective in the upper troposphere than in the lower troposphere.

The mechanical energy produced and dissipated in a hurricane is approximately 20% of the heat taken from the sea. Landsea's hurricane FAQ web site estimates the upward heat flux in an average size hurricane at 600 TW, and the mechanical energy required to maintain the circulation at 3 TW for an efficiency of 0.5%. The single restriction model shows that the efficiency is 20%, forty times higher than the efficiency based on estimates of the mechanical energy required to maintain the circulation. The mechanical energy produced and dissipated in Landsea's average size hurricane would be 120 TW, 40 times more than the total energy produced by humans. It is estimated that 50 to 80% of the energy of hurricanes is the result of heat received from air-sea interaction at reduced pressure near the eyewall; the remainder is due to heat received from the sea at near normal surface pressure either prior to or during the hurricane.

The maximum kinetic energy of the wind in a hurricane is also a poor indication of how much mechanical energy is produced. The Dvorak relationship between central pressure and maximum wind, see Holland (1997), fig. 4, shows that the maximum kinetic energy of the air is about 40% to 50% of the kinetic energy produced during isentropic expansion from normal surface pressure to eyewall pressure. There is a thermodynamic relationship between upward heat flux and mechanical energy produced, but there is no definite relationship between upward heat flux and maximum kinetic energy because the maximum kinetic energy depends on the nature of the dissipative process. Conversion efficiency is difficult to calculate from observation of kinetic energy or from estimate of the energy required to overcome frictional dissipation because dissipation is not as well understood as thermodynamic energy conversion.

Michaud (2000) used a heating increment technique to show that 20 to 25% of the heat transported upward in tropical cyclones is converted to mechanical energy. Here is an example of the technique: the work increment of $1923\,\mathrm{J\,kg^{-1}}$ between cases 1 and 2 of Table 2 is 31% of the energy received increment. Taking the temperatures of the hot and cold source as 300 and 206 K respectively gives a Carnot efficiency of 31%. The incremental energy received as heat is $4423\,\mathrm{J\,kg^{-1}}$ (88330–83807); and the incremental energy received as additional dissipated work is $1923\,\mathrm{J\,kg^{-1}}$, $1923/(1923+4423) = 0.30$. The increment technique gives similar results when applied between cases 1 and 3 of Table 2.

Emanuel (1997) used a thermodynamic approach to calculate hurricane intensities, his method implicitly recognizes that mechanical energy is produced and dissipated. Emanuel based his intensity calculations on sea-air heat transfer and Carnot efficiency calculated from SST and outflow temperature. The sensitivity of the Emanuel method to SST is $0.6\,\mathrm{kPa\,K^{-1}}$ according to Shade and Emanuel (1999), and $1.0\,\mathrm{kPa\,K^{-1}}$ according to Willoughby and Black (1996). Shade (2000) argued that the sensitivity of the Emanuel model is actually $1.5\,\mathrm{kPa\,K^{-1}}$ because the intensity is sensitive to outflow temperature which is correlated to SST. It is not clear whether Emanuel's sensitivity is based on the eyewall SST or the pre-Storm SST. Despite the difference in sensitivity to SST, MPI's calculated with the Emanuel's and with the Holland's methods are quite close, see Henderson-Sellers et al. (1998), fig. 3. The Emanuel method only considers the heat transferred from the sea to the air during the storm; it does not consider the contribution of the heat content of the surface air prior to the storm.

5. Conclusion

The determination of which variables are conserved is a key consideration in applying the total energy equation. Once the conserved variables have been established the missing variables can easily be found. In reversible case A, entropy and mixing ratio are conserved during process 1–2; in irreversible case B, enthalpy (h) and mixing ratio are conserved during process 1–2. Entropy (s), static energy (μ) and mixing ratio (r) are all conserved during process 2–3 in all three cases. P_2 in Case A can be determined because mixing ratio and entropy at state 2 are known. P_2 in Case B can be determined because mixing ratio and enthalpy at state 2 are known. P_2 in Case C can be determined because relative humidity and temperature in state 2 are known.

The sensitivity of MPI to surface air temperature and humidity was calculated using the total energy equation and the results of the calculations were compared with observations and with other models. The *intensity* of tropical cyclones was shown to be primarily a function of SST because the rising air approaches equilibrium with the water, and because the elevation of geo-potential surfaces at the top of the troposphere in subtropical latitudes is essentially constant. It is not difficult to estimate the sensitivity of MPI to SST because the pressure and temperature ranges are very limited; the sensitivity must be close to $2\,\mathrm{kPa\,K^{-1}}$ because MPI has a range of 98 to 86 kPa, while SST has a range of 26 to 31 °C. The TEE model uses thermodynamic principles to show that the sensitivity of eyewall intensity to SAT is approximately $2.9\,\mathrm{kPa\,K^{-1}}$; that the sensitivity of eyewall intensity to eyewall SST is 1.5 to $2.5\,\mathrm{kPa\,K^{-1}}$ because SAC increases with intensity, and that the sensitivity of eyewall intensity to SST prior to the storm is 1.0 to $2.0\,\mathrm{kPa\,K^{-1}}$ because of SST reduction during the storm. The TEE method not only predicts the intensity, it provides an explanation for the energy of the wind and for the surface pressure reduction.

Mechanical energy is produced when heat is transported upward by convection from the sea –

the hot source, to the upper troposphere – the cold source. The kinetic energy dissipated in hurricanes is approximately 20% of the heat removed from the sea because the efficiency is essentially constant. The efficiency is essentially constant because the temperatures of the hot and cold source are essentially constant. The discrepancies between various estimates of the energy produced in hurricanes needs to be resolved. The total energy equation together with steady state ideal processes such as those of Fig. 1 could help resolve the discrepancy between estimates of the efficiency of the hurricane process. The TEE method for calculating hurricane intensity is far simpler than commonly used methods and can help understand the energy conversion process.

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