Note on: The energy deficit of 20 to 30 W m^{-2} observed in climate model.

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Abstract

The *integrated-enthalpy* deficits observed in numerical models provide an unexpected proof that the mechanical energy produced and dissipated in the atmosphere is around 25 W m^{-2}. The heat to work conversion efficiency of the atmosphere is shown to be approximately *one sixth* because the average temperatures at which the atmosphere receives and gives up heat are ≈300 K and ≈250 K respectively. The conversion efficiency is essentially a function of the temperatures at which heat is received and given up. The mechanical energy produced and dissipated is in the 20-30 W m^{-2} range since the upward heat flux near the bottom of the atmosphere approximately 150 W m^{-2}.

1. Introduction

Conservation of energy requires that the energy of a closed system with no external energy input be constant, in other words the change in the energy of a closed thermodynamic system must equal the net energy input to the system. The *total-energy* of a closed system is the sum of its enthalpy and its kinetic energy. Emanuel and Zivkovic-Rothman (1999, EZ hereafter) noted that the observed energy fluxes in the TOGA COARE intensive flux array would result in a gradual decrease in *integrated-enthalpy* with no long term change in the kinetic energy. EZ pointed out that the decrease in *integrated-enthalpy* is a serious error, which corresponds to an average shortage in the net energy input of 22 W m^{-2}. They noted that the predicted column enthalpy will have serious errors irrespective of the performance of the convection scheme. EZ adjusted the observed infrared radiation at the top of the atmosphere downward to avoid a gradual decrease in *integrated-enthalpy*. The EZ observation is not unique, in the model of Wu at al. (1998) the outgoing long wave radiation is 35 W m^{-2} lower than observed.

Michaud (1996) showed that the work produced and dissipated during upward heat convection is approximately 25 W m^{-2}. The enthalpy deficit observed by EZ could be the result of either under-estimating mechanical energy production or of neglecting mechanical energy dissipation. Ignoring the fate of the mechanical energy is equivalent to letting the mechanical energy escape from the system. The *integrated-enthalpy* deficit observed by EZ provides unexpected proof that the mechanical energy produced and dissipated in the atmosphere is around 25 W m^{-2}. This comment also shows that approximately one sixth of the upward convective heat flux at the bottom of the atmosphere is converted to mechanical energy which usually dissipates rapidly.

2. Numerical illustration

Upward heat convection in the atmosphere is an irreversible process; any work produced during the process must be dissipated within the system since no work leaves the system. The closed thermodynamic steady-state system shown in Fig. 1 illustrates the effect of neglecting the work produced during upward heat convection. The system consists of an insulated air column covered by a piston exerting a constant pressure; work production (*W*) is represented by a turbine and work dissipation is represented by a paddlewheel. In the reversible process of Fig. 1a, the heat leaving the system (Q_o) is less than the heat entering (Q_i) the system because 25 W m^{-2} leave the system as mechanical energy. In the irreversible process of Fig. 1b, the heat leaving the system is equal to the heat entering the system because the mechanical energy is dissipated within the system and ultimately increases the temperature of the air. In the reversible process, the external entropy input ($S_{e\text{-}i}$) is equal the external entropy output ($S_{e\text{-}o}$) and entropy is conserved. In the irreversible process, $S_{e\text{-}i}$ is less than $S_{e\text{-}o}$ and entropy must be produced internally within the system (S_i) to make up the difference. The internally generated entropy of 0.1 W m^{-2} K^{-1} could be produced by dissipating 25 W m^{-2} at 250 K, or 30 W m^{-2} at 300 K, or some equivalent intermediate combination, see Michaud (1996).

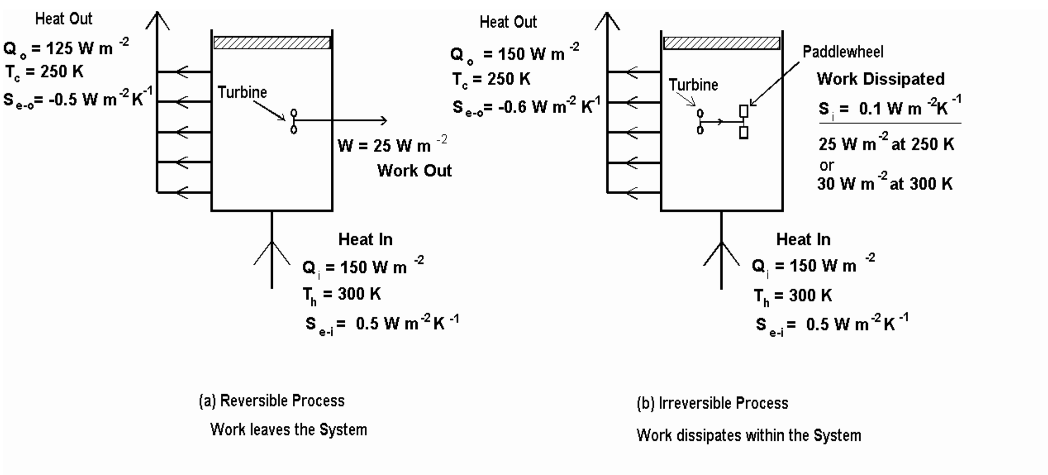

Fig. 1 In reversible process (a) the heat out is less than the heat in because work is ignored, which is equivalent to letting the work leave the system. In irreversivble process (b) the heat out is equal to the heat in because the work is dissipated within the system.

Fig. 2 illustrates the process with a numerical example. The column of dry air initially has a uniform potential temperature, θ = 300 K, and extends from P_1 = 100 kPa where T_1 = 300 K, to P_2 = 52.83 kPa where T_2 =250 K. $T_2/T_1 = (P_2/P_1)^{Ra/Cpa}$ = 5/6, where R_a/C_{pa}= 2/7, where R_a and C_{pa} are the gas constant and the specific heat at constant pressure of dry air, and where $C_{pa} \approx$ 1000 J kg^{-1}. The air column has an adiabatic lapse rate of $a = g/C_{pa}$ = 9.75 K km^{-1} and a height of (300-250)/a = 5126 m.

The unit mass of air at the bottom of the column is heated by 6 K, then raised isentropically to the top of the column, and then cooled to return the column to its initial condition. The temperature of the bottom unit mass is increased from 300 K to 306 K by the addition of 6 C_{pa}= 6000 J of heat. The temperature of the θ=306 K unit mass after it is raised isentropically to the top of the column is T = 255 K, 5/6 of 306 K. Restoring the system to its initial state requires that the temperature of the unit mass be decreased from 255 K to 250 K by the removal of 5 C_{pa}= 5000 J of heat.

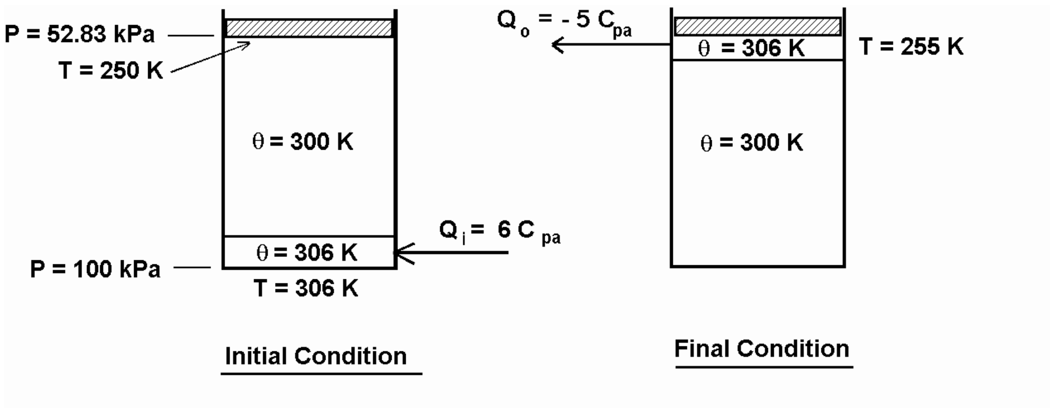

Fig. 2 Reversible process consisting of a column of dry air with uniform potential temperature. The heat that needs to be removed to restore the initial condition is less that the heat supplied because some of the heat is converted to work.

The heat that has to be removed to restore the original condition is less than the heat supplied because 1000 J of heat was converted to mechanical energy and because the fate of this mechanical energy was not considered. The mechanical energy may be low-grade turbulence, but ignoring its fate results in a significant error in the energy balance. Dissipating the work anywhere within the system would keep the *integrated-enthalpy* constant. Dissipating the work in a single unit mass of air would increase its temperature by 1 K. Dissipating the work uniformly in a column of air of unit area containing approximately 5000 kg of air would increase the average temperature of the column by 0.0002 K. Dissipating the work in the raised unit mass of air would increase its final temperature from 255 to 256 K. Restoring the system to its initial state requires the removal of 6000 J when the work is dissipated within the system, but only 5000 J when the work is ignored or allowed to leave the system.

The heat to work energy conversion efficiency (n) during upward heat convection is given by the Carnot efficiency equation, $n=1-T_c/T_h$, where T_h and T_c are the average temperature at which heat is received and given up by the system, see Renno and Ingersoll (1996). Renno and Ingersoll (1996) state: "The essential feature of a heat engine is that heat must be absorbed by the working fluid at a higher temperature than it is rejected". The systems of Fig. 1 and Fig. 2 were designed so that the temperatures at which heat is received and given up correspond roughly to the temperatures at which heat is received and given up in the atmosphere. The temperatures at which the heat is received and given up are 300 and 250 K respectively in both cases, therefore the efficiency is 16.7% (*one sixth*) in both cases. The temperature at which the heat is given up is the average temperature of the column in Fig 1, and the temperature at the top of the column in Fig. 2. Taking the average convective heat flux in the lower atmosphere as ≈150 W m^{-2}, the average work produced in the atmosphere is ≈25 W m^{-2}. The efficiency is independent of whether the heat is carried as latent or as sensitive heat, Michaud (2000). The ideal system of Fig. 2 was used to illustrate the process. Fig. 1 is more representative of the atmosphere where the presence of water reduces the lapse rate and where the radiative cooling which occurs at all levels is compensated for by subsidence warming.

3. Discussion

Van Ness (1969, page 27) states: “The reversible process is the only one for which we can readily do calculations. The alternative is likely to be that we do no calculations at all”. The temperature of atmospheric updrafts is almost invariably calculated assuming isentropic expansion because where and how much work is dissipated is not usually known. Updraft temperatures calculated assuming reversible isentropic expansion are an excellent approximation of the expansion temperature when the work is not dissipated within the updraft, and a good approximation when the work is dissipated within the updraft, 255 K versus 256 K in the above example. The temperature of the rising air decreases by the adiabatic lapse rate, 50/5126=9.75 K km^{-1} in the reversible process, by 49/5126=9.56 K km^{-1} in the irreversible process where the starting temperature of 306 K, and would decrease by 48/5126 = 9.36 K km^{-1} in an irreversible process with a starting temperature of 312 K. The reversible dT/dz is the only which can readily be calculated because the irreversible dT/dz depends on how much work is produced and where it is dissipated. The work produced during upward heat convection is usually ignored, but its ultimate fate must be considered in order to obey the law of conservation of energy.

The potential temperature of the updraft is constant in the reversible process and increases in the irreversible process. The potential temperature of the rising air at the upper level is 306 K for the reversible process and 307.2 K for the irreversible process when the work is dissipated within the updraft. The reason for the *integrated-enthalpy* deficit observed by EZ is that the final temperature of updrafts rising irreversibly is slightly higher than the final temperature of updrafts rising reversibly. The rate of change of temperature with pressure dT/dP is higher for reversible updrafts than for irreversible updrafts. The final temperature of updrafts rising reversibly is lower than the final temperature of updrafts rising irreversibly. Therefore the radiative heat flux at the top of the troposphere must be reduced to avoid a gradual decrease in the integrated-enthalpy when reversible updrafts are used.

Van Ness (1969, pages 15-25) pointed out that reversibility requires mechanical equilibrium. Work dissipation can occur as a result of lack of mechanical equilibrium and not only because of viscous friction. In the absence of a force to resist the force of expansion, the mechanical energy readily dissipates. Michaud (1995) stated that the work is actually produced and dissipated. The statement could be challenged; engineers only consider work to be have been produced once it becomes shaft work. Whether the work is actually produced or not, a quantity of mechanical energy equivalent to the mechanical energy which would be produced in a reversible process must be dissipated in order to conserve energy.

Irreversibility does not mean that mechanical energy is not produced, irreversibility mean that the mechanical energy is dissipated before it can be captured and become shaft work. There are irreversible processes such as heat conduction in a solid where there is no work production and where there is only a potential to do work, but in a process where heat is carried in a gaseous phase and where the temperature at which heat is received is higher than the temperature at which heat is given up work must be produced.

Emanuel and Bister (1996) estimated that 20% of the internally generated entropy can be produced by processes other than the dissipation of mechanical energy, processes such as: mixing, and evaporation. There are several processes which can produce entropy, but there are only two processes which can increase the total energy of the closed system; they are the addition of heat or work from outside the system. There is no external energy input during the lifting process, therefore the only process which can prevent the decrease in the total-enthalpy the system is the dissipation of mechanical energy; the mechanical energy must be produced within the system since there is no external work input.

The assumption that air masses expand isentropically is the root cause of the decrease in the integrated-enthalpy. Reversible isentropic processes are used to calculate the temperature of updrafts, but the overall upward heat convection process is irreversible. No work is taken out of the system therefore the work must be dissipated within the system. The integrated-entropy of the system is conserved in a reversible process. The integrated-enthalpy of a system is conserved in an irreversible process. The entropy of the updraft is conserved in reversible updraft processes. The *static-energy* of the updraft, the sum of its enthalpy and potential energy, is conserved in an irreversible updraft process where the work is dissipated within in the updraft. Exactly where the mechanical energy is dissipated is not known, but dissipating the work within the updraft is a good starting point since it is likely that most of the dissipation occurs within the rising air. Updraft temperatures calculated based on constant updraft static-energy process are better than updraft temperatures calculated based on constant entropy process because they result in conservation of energy.

The mechanical energy produced in the atmosphere is commonly believed to be in around 2 W m^{-2}, Pexioto and Oort (1992). One of the reason for not realizing how much mechanical energy is produced is that the energy can be dissipated as it is produced. While the mechanical energy produced in raising the kilogram of air is 1000 J kg^{-1}, corresponds to a velocity of 45 m s^{-1}; the kinetic energy of the kilogram of ascending air rising at a typical upward velocity of 2 m s^{-1} is only 2 J kg^{-1}. The fact, that the work is dissipated as it is produced, makes it difficult to determine how much kinetic energy is produced from observations. The kinetic energy generally does not last long enough to be observable. How much velocity is produced depends on how the flow is organized; the maximum velocity produced by a mechanical energy of 1000 J kg^{-1} can approach 45 m s^{-1} in well organized flow, but can be under 1 m s^{-1} in unorganized flow. Michaud (1999 and 2000) proposed that mechanisms such as the solar chimney or an equivalent vortex could be used to organize the flow so that the mechanical energy can be concentrated and captured before it dissipates.

The work required to maintain the general circulation is probably very small compared to the energy produced. The work required to maintain a steady wind of 20 m s^{-1} in the whole troposphere is well under 1 W m^{-2} because a steady flow requires little shear. Rather than asking why is the kinetic energy of the wind so high, one should be asking why is the kinetic energy of the wind so low. The answer is that most of the work dissipates in a very short time. In unorganized cumulus, the work is dissipated in the vicinity of the updraft. In organized convection such as convective vortices, the work is transferred downward and takes a little longer to dissipate. The potential energy of water flowing down a slope dissipate as the water trickles down; the mechanical energy produced during unorganized upward heat convection dissipates as heat trickles up. Observable kinetic energy is a poor indication of how much mechanical energy is produced because mechanical energy can dissipate as it is produced.

Margules (1905) was the first person to use the piston covered column systems to calculate the mechanical kinetic energy produced when air masses are re-arranged. He reasoned that the missing internal energy had to be transformed into mechanical energy. EZ succeeded in observing the energy deficit and concluded that the measured upward heat flux at the top of the atmosphere was too high. Margules was correct; Randall and Wang (1992) recently used the Margules approach to calculate the mechanical energy produced when a 2.5 kPa layer is raised from the bottom to the top of the troposphere. Bister and Emanuel (1998) showed that the dissipation of mechanical energy can increase hurricane wind velocity by about 20%, but did not realize that neglect of work dissipation is also responsible for the decrease in the *integrated-enthalpy*.

4. Conclusion

This comments brings out key features of the atmospheric process. The heat to work conversion efficiency of the atmosphere is approximately *one sixth* because the average temperatures at which the atmosphere receives and gives up heat are ≈300 K and ≈250 K respectively. The conversion efficiency is essentially a function of the temperatures at which heat is received and given up. The mechanical energy produced and dissipated is in the 20-30 W m^{-2} range because the upward heat flux near the bottom of the atmosphere is approximately 150 W m^{-2}. The average efficiency and the total mechanical energy produce are constrained to narrow ranges of values despite the variability of the atmospheric process.

The *one-sixth rule* is valid whether the heat is transported as sensible or latent heat, and whether the heat is radiated to space at the latitude where it is received or in higher latitude. A direct corollary of the *one-sixth rule* is that: there will be an *integrated-enthalpy* deficit of 20 to 30 W m^{-2} if the temperature of updrafts is calculated based on reversible expansion and if the fate of the mechanical energy is ignored.

The 22 W m^{-2} deficit found by EZ provides an unexpected proof that the mechanical energy produced and dissipated in the atmosphere is approximately 25 W m^{-2}. Neglecting the disposition of the mechanical energy has little effect on expansion temperature, but has a significant effect on the energy budget. The mechanical energy can be too short-lived to be observable, but the fact that mechanical energy was dissipated can be inferred from the *integrated-enthalpy* deficit. Why is mechanical energy important if dissipates so rapidly? Mechanical energy is important not only to get energy balance, but also because it might be possible to devise low dissipation upflow processes and capture the work which is normally dissipated.

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