# On Hurricane Energy

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## Abstract

Warm sea water is the energy source for hurricanes. Interfacial sea-to-air heat transfer without spray ranges from 100 W m$^{-2}$ in light wind to 1000 W m$^{-2}$ in hurricane force wind. Spray can increase sea-to-air heat transfer by 2 orders of magnitude and result in heat transfers of up to 100,000 W m$^{-2}$. Drops of spray falling back in the sea can be 2 to 4 °C colder than the drops leaving the sea thus transferring a large quantity of heat from sea to air. The heat of evaporation is taken from the sensible heat of the remainder of the drop; evaporating approximately 0.3% of a drop is sufficient to reduce its temperature to the wet bulb temperature of the air. The heat required to evaporate hurricane precipitation is roughly equal to the heat removed from the sea indicating that sea cooling is due to heat removal from above and not to the mixing of cold water from below. The paper shows how case studies of ideal thermodynamics processes can help explain hurricane intensity.

## 1. Introduction

Several methods of establishing hurricane maximum potential intensity (MPI) have been proposed; the two leading methods are E-MPI, Emanuel (1986) and H-MPI Holland (1997). Persing and Montgomery (2003) and Bell and Montgomery (2008) noted that E-MPI has gained much acceptance, but that E-MPI can not account for observed super-intensities. E-MPI is based on the heat transfer from sea to air multiplied by a Carnot efficiency calculated using the temperatures of the sea for the hot source temperature and the temperature at the top of the hurricane for the cold source temperature. H-MPI is based on the pressure at the bottom of a column of air approaching equilibrium with the underlying water at the reduced eyewall pressure. Camp and Montgomery (2001) reviewed the Holland and Emanuel MPI methods; they noted that H-MPI tends to overestimate the intensity of strong hurricanes while E-MPI tends to underestimate the same. Persing and Montgomery (2003) suggested that a new MPI formulation from first principles is required; this paper shows that thermodynamic case studies of idealized processes could provide such a formulation.

Maximum hurricane heat fluxes calculated using the Dalton heat transfer coefficient are in the order of 1000 W m$^{-2}$, Black et al. (2007), Trenberth et al. (2007). Explaining hurricane precipitation and the observed ocean cooling requires heat fluxes in the order of 100,000 W m$^{-2}$. The prevailing view is that maximum hurricane sea to air heat fluxes are under 1000 W m$^{-2}$. Trenberth et al. (2007) point out that the heat fluxes used in hurricane models require that the circulation reach out to 3 to 5 times the precipitation area to gather the moisture; one would have to integrate out to 1600 km to get a rough energy balance. Josey et al. (1999) point out that many studies have found that heat fluxes calculated using the Dalton coefficient result in a mean ocean heat gain of 30 W m$^{-2}$. D'Asaro et al. (2007) show that the ocean cooling takes place close to the hurricane eye and that it is concentrated to the right of the hurricane track but maintain that the cooling is almost entirely due to vertical mixing and not to air-sea heat fluxes. Shay et al. (2000) estimated that only 10 to 15% of the ocean cooling is due to surface heat flux and that the remainder is due to mixing of cold water from below. Andreas and Emanuel (2001) considered the effect of spray and concluded that spray can provide a significant fraction of the sea to air heat flux. Their maximum heat fluxes are under 5000 W m$^{-2}$; their estimate of the quantity of spray may have been low.

High quality data for 2003 hurricane Isabel recently became available, Montgomery et al. (2006). The Isabel data comprised multiple dropsondes in four areas: eye, eyewall, outer core, and distant environment. Isabel was a large category 5 hurricane (wind speed > 67 m s$^{-1}$) during an intense observation period (IOP) which took place on 13 September 2003 between 1600 and 2300 UTC when Isabel was located 1300 km east of Puerto Rico and was essentially at steady state. Bell and Montgomery (2008) noted that the observed wind of 76 m s$^{-1}$ is well in excess of the E-MPI of 57 m s$^{-1}$.

Michaud (2000 and 2001) showed that MPI can be calculated by applying the total energy equation (TEE) to steady state ideal processes. This paper applies the TEE method to the new Isabel data and shows that it yields MPI's that are consistent with observations. Section 2 calculates MPI from eyewall air temperature and relative humidity. Section 3 examines the effect of spray on eyewall air relative humidity and temperature. Section 4 looks at cumulative hurricane sea to air heat transfer and at the contribution of ocean heat content.

## 2. Effect of eyewall air temperature and humidity on MPI – Lifting process 3-4

The TEE method of analysis is based on the steady state ideal open thermodynamic process shown in Fig. 1. State 1 corresponds to distant environment surface air. State 3 corresponds to eyewall surface air. State 4 is the level of neutral buoyancy. The air in state 3 conditions approaches equilibrium with the sea surface temperature (SST) at the reduced surface pressure ($P_3$). Entropy ($s$) is conserved in reversible adiabatic expansion processes 1-2 and 3-4. Process 1-2 is essentially an isentropic expander whose work is equal to the decrease in the enthalpy ($h$). An isentropic expander can be a turbine as shown in Fig. 1 or a reciprocating piston and cylinder machine. Process 2-3 represents the sea-to-air heat and mass transfer wherein enthalpy is transferred from water to air; the enthalpy gain of the air in process 2-3 is equal to the enthalpy loss of the water in process 7-8. Process 3-4 represents the upward flow process in the eyewall; process 3-4 is adiabatic and can be considered to take place in an insulated vertical tube.

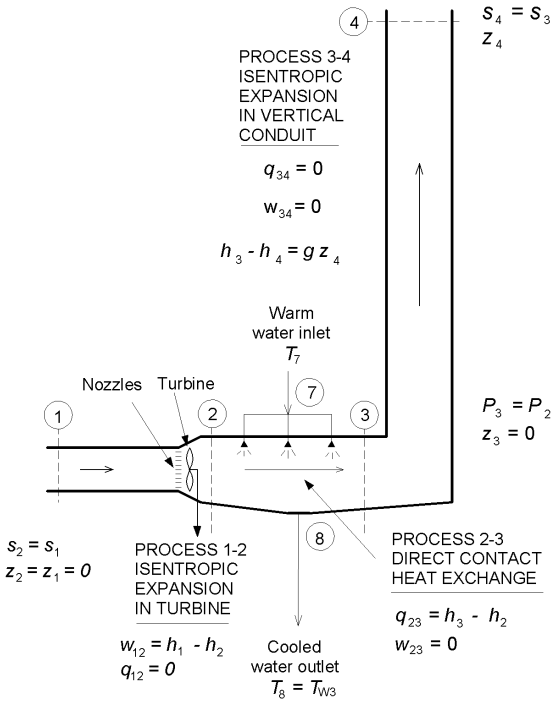

**Fig. 1 Hurricane Steady-State Ideal Process.** State 1 ambient surface air. State 2 air at reduced pressure prior to the mixing process. State 3 air at reduced pressure approaching equilibrium with SST after the mixing process. State 4 level of neutral buoyancy. Process 1-2 isentropic expansion. Process 2-3 isobaric-isenthalpic mixing. Process 3-4 isentropic expansion in upward flow conduit. State 7 water at eyewall SST. State 8 water at State 3 wet bulb temperature

The total energy equation is:

$$W = Q - \Delta h - \Delta gz$$

where $W$ is work, $Q$ is heat, $h$ is enthalpy all in Joules per kilogram of dry air; and where $g$ is the acceleration of gravity and $z$ is height. Enthalpy ($h$) includes the contribution of the water content of the air in any phase. A kilogram of dry air contains $(1 + r)$ kilogram of substance, where $r$ is the mixing ratio in kilogram of water per kilogram of air. Since the enthalpy term includes the contribution of the water content of the air, the $gz$ term needs to be multiplied by $(1 + r)$. Total work is the work per unit mass of dry air multiplied by the flow of dry air. Capital $W$ and $Q$ are used in this article for specific work for readability since work and heat are always per unit mass.

The total energy equation is applicable to each individual process and to the overall process; Fig. 1 shows the terms applicable to each process. There is no change in height in processes 1-2 and 2-3; there is no heat exchange in adiabatic processes 1-2 and 3-4; and there is no work in process 2-3. Idealizing processes to establish their maximum performance is a standard thermodynamic practice; velocities and frictional losses are initially assumed to be negligible. The kinetic energy of the air is considered to be negligible except at the exit of the turbine nozzles which is immediately captured by the turbine blade. Ideal reversible processes are the only ones for which thermodynamic analysis is readily possible. In order for a process to be reversible the work must be removed from the system to prevent its dissipation. Once the reversible process is understood, irreversible processes can readily be analyzed. For example process 1-2 can be made irreversible by reducing the efficiency of the turbine; process 1-2 can be changed from a constant entropy process to a constant enthalpy process by reducing turbine efficiency from 100 to 0% or by replacing the turbine with a restriction.

*This paper uses eyewall pressure as the primary measure of MPI.* MPI sometimes refer to maximum wind speed, to minimum eye pressure, or to maximum eye pressure reduction. The selected MPI parameter is not important since the intensity parameters move in unison. The TEE method calculates the pressure ($P_3$) at the base of a column of warm humid air rising to its level of neutral buoyancy ($P_4$). MPI calculation is based on the realization that $P_3$ is the pressure for which the isentropic work of expansion from $P_3$ to $P_4$ equal to $h_3 - h_4$ is equal to the increase in potential energy in process 3-4. Without the use of the reversible process it is difficult to see that $P_3$ is the pressure for which: $(h_3 - h_4) - (1 + r_3)\ gz_4 = 0$. $P_3$ is calculated by iteration; interpolation is used to determine the value of $P_3$ for which the net work in process 3-4 is zero. $P_3$ calculation is illustrated with an example in the Appendix. The simulation basically calculates the minimum pressure $P_3$ that can be produced by raising air with temperature $T_3$ and relative humidity $U_3$. The TEE method is simple and direct; one or two iterations are sufficient to calculate $P_3$. Once $P_3$ is known expander work $W_{12}$ is readily calculated by expanding ambient surface air isentropically from $P_1$ to $P_3$.

The TEE method was applied to the Isabel data. Table 1 shows results for three cases. Thermodynamic properties are per unit mass of dry air and are consistent with those of Ooyama (2001) and Bechtold (2009). Case 1 state 1 temperature $T_1$ of 27.8 °C and relative humidity $U_1$ of 75% the distant environment

**Table 1 Hurricane intensity calculation based on hurricane Isabel data**

Upper section: Pressure and work calculation. Lower section spray to air ratio calculations. Parameters: $P$ pressure (kPa), $T$ temperature (°C), $U$ relative humidity (%), $r$ mixing ratio (g-water per kg dry air), $h$ enthalpy (J $kg^{-1}$), $s$ entropy (J $K^{-1}$ $kg^{-1}$), $T_w$ wet bulb temperature (°C), $W$ specific work (J $kg^{-1}$) Q specific heat (J $kg^{-1}$), $v$ velocity (m $s^{-1}$), $n$ efficiency (%), $M$ water spray to air mass ratio (kg-water per kg air). Subscripts: r process 1-2 reversible and isentropic, I process 1-2 irreversible and isenthalpic. State 1 air properties: $P_1 = 101.1$ kPa, $T_1 = 27.8$ °C, $U_1 = 75\%$, $r_1 = r_2 = 17.87$ g $kg^{-1}$, $h_1 = 73530$ J $kg^{-1}$, $s_1 = s_2 = 256.5$ J $K^{-1}$ $kg^{-1}$, $T_{1w} = 24.4$ °C. Key inputs and results are shown in bold.

| **Processes 1-4** | **Case 1** | **Case 2** | **Case 3** |
|---|---|---|---|
| $P_2 = P_3$ (kPa) | 98.63 | **96.90** | **94.26** |
| $T_2$ (°C) | 25.69 | 24.19 | 21.93 |
| $U_2$ (%) | 83.1 | 89.5 | 100.2 |
| $T_{2w}$ (°C) | 23.5 | 22.9 | 21.9 |
| $h_2$ (J $kg^{-1}$) | 71340 | 69780 | 67370 |
| | | | |
| $T_3$ (°C) | 25.69 | **24.5** | **25.5** |
| $U_3$ (%) | 83.1 | **97** | **97** |
| $r_3 = r_4$ (g $kg^{-1}$) | 17.87 | 19.74 | 21.60 |
| $T_{3w}$ (°C) | 23.5 | 24.1 | 25.1 |
| $h_3$ (J $kg^{-1}$) | 71340 | 74870 | 80650 |
| $s_3 = s_4$ (J $K^{-1}$ $kg^{-1}$) | 256.5 | 273.7 | 301.3 |
| | | | |
| $P_4$ (kPa) | 12.0 | 12.0 | 12.0 |
| $T_4$ (°C) | -74.22 | -70.25 | -64.21 |
| $h_4$ (J $kg^{-1}$) | -83270 | -80030 | -74530 |
| $z_4$ (m) | 15500 | 15500 | 15500 |
| $gz_4 (1 + r_3/1000)$ | 154610 | 154900 | 155180 |
| | | | |
| **Pressure Reduction** | | | |
| $\Delta P_{12}$ (kPa) | 2.47 | **4.20** | **6.84** |
| | | | |
| **Heat, Work and Velocity** | | | |
| $Q_{23} = h_3 - h_2$ (J $kg^{-1}$) | 0 | 5090 | 13280 |
| $W_{12} = h_1 - h_2$ (J $kg^{-1}$) | **2190** | **3750** | **6160** |
| $v = (2\ W_{12})^{0.5}$ (m $s^{-1}$) | 66.2 | **85.8** | **109.8** |
| | | | |
| **Efficiency** | | | |
| $n$ (%) $= \Delta W_{12}/\Delta Q_{23r}$ | n/a | 30.6 | 29.9 |
| $n$ (%) $= 1 - T_4/T_3$ | n/a | 31.8 | 30.0 |
| | | | |

| **Mixing Process 2-3 & 7-8 for Case 2** | $r_2 = 17.87$ | $r_2 = 17.87$ | | $r_2 = 2.0$ | |
|---|---|---|---|---|---|
| $T_3$ (°C) | 24.5 | 24.5 | 24.5 | 24.5 | 24.5 |
| $T_7$ (°C) | **25** | **26** | **29** | **26** | **29** |
| $T_8$ (°C) | 24.1 | 24.1 | 24.1 | 24.1 | 24.1 |
| **Process 1-2 isentropic** | | | | | |
| $T_{2r} = T_1$ (°C) | 24.2 | 24.2 | 24.2 | 24.2 | 24.2 |
| $M_{7r}$ (kg-water kg-air$^{-1}$) | 1.35 | 0.62 | 0.24 | 5.58 | 2.14 |
| $Q_{23r} = h_3 - h_2$ (J $kg^{-1}$) | 5090 | 5090 | 5090 | 5090 | 5090 |
| **Process 1-2 isenthalpic** | | | | | |
| $T_{2i} = T_1$ (°C) | 27.8 | 27.8 | 27.8 | 27.8 | 27.8 |
| $M_{7i}$ (kg-water kg-air$^{-1}$) | 0.31 | 0.14 | 0.056 | 5.12 | 1.96 |
| $Q_{23i} = h_3 - h_1$ | 1340 | 1340 | 1340 | 1340 | 1340 |

surface air taken from Fig. 4 of Montgomery et al. (2006). In case 1, there is no water spray; pressure ($P_3$) is 98.63 kPa, work ($W_{12}$) is 2190 W kg$^{-1}$ corresponding to a velocity of 66.2 m s$^{-1}$. In case 2, Isabel eyewall air temperature and humidity are used for state 3 conditions. The case 2 eyewall air temperature and relative humidity of 24.5 °C and 97% were also taken from Fig. 4 of Montgomery et al. (2006). In case 2, the calculated eyewall pressure ($P_3$) is 96.9 kPa and the work $W_{12}$ is 3750 W kg$^{-1}$ corresponding to a velocity ($v$) of 86 m s$^{-1}$. In case 3, the eyewall air temperature is increased by 1 °C from 24.5 to 25.5 °C. In case 3, $P_3$ is 94.3 kPa, and $W_{12}$ is 6160 W kg$^{-1}$ corresponding to a velocity of 110 m s$^{-1}$. Hurricane Isabel dropsondes measured sustained wind speed of and peak wind speeds of 77 and 110 m s$^{-1}$ respectively, Montgomery et al. (2006).

The atmosphere is an engine wherein heat is partly converted to work. In Table 1, the heat to work conversion efficiency of approximately 30% is calculated in two ways to provide an independent check:

1. The Carnot efficiency using $T_3$ for the hot source temperature and $T_4$ for the cold source temperature.
2. Incremental efficiency, defined as the increment in work per increment in heat, is used instead of efficiency because the CAPE of the distant ambient air (case 1) is 2190 W kg$^{-1}$ and not zero. In case 2, adding 5090 W kg$^{-1}$ of heat in process 2-3 increases process 1-2 work from 2190 to 3750 W kg$^{-1}$ resulting in a work increment of 1560 J kg$^{-1}$ and an incremental conversion efficiency of 30.7%, (1560/5090).

Sadi Carnot realized that there is a potential for producing work whenever heat flows from a hot source to a cold source provided the process is carried our reversibly, which can be achieve with Carnot engines. He wrote: "to the flow of heat is due all movement including wind and precipitation". *There is a potential for producing work when heat flow upward in the atmosphere.* Michaud (2000) showed that the efficiency of the open process of Fig. 1 is the same as that of a corresponding close cycle.

Pressure ($P_3$), work ($W_{12}$) and velocity ($v$) are very sensitive to air temperature ($T_3$). The sensitivity of eyewall pressure to $T_3$ is -2.64 kPa K$^{-1}$. The sensitivity of work of convection $W_{12}$ to $T_3$ is 2410 J kg$^{-1}$ K$^{-1}$. The sensitivity of velocity to $T_3$ is 24 m s$^{-1}$ K$^{-1}$. Fig. 2 shows how eyewall pressure is affected by eyewall air temperature and relative humidity. The isobaric curves show the combinations of state 3 temperatures and relative humidity capable of producing a given surface pressure $P_3$; case 1, 2, and 3 state 3 conditions are marked with crosses. Simulation results can be used to determine the sensitivity to a wide range of parameters. Simulators results can easily be checked; examination of Table 1 data shows: that entropy is conserved in processes 1-2 and 3-4; that the net work in process 3-4 is zero; and that the enthalpy increase of the air in process 2-3 is equal to enthalpy decrease of the water in process 7-8.

The level of neutral buoyancy (LNB) for hurricanes is typically between 15 and 10 kPa. The 15500 m elevation of the 12 kPa in Table 1 is based on the Jordan (1958) mean hurricane season sounding for the West Indies. Geopotential heights in the tropical upper troposphere are governed by the earth's energy budget and are fairly constant. $P_3$ is not very sensitive to $P_4$ because the buoyancy of the raised air whether positive or negative is small near the level of neutral buoyancy. The LNB's in cases 2 and 3 are 12 kPa and 11 kPa respectively. A LNB of 12 kPa was used for all three cases in Table 1 for ease of comparison. Additional iterations can be used to find the level of neutral buoyancy by searching for the pressure ($P_4$) which maximizes the work. The LNB could be close to 12 kPa in most hurricanes because the temperature of rising air decreases rapidly (by approximately 5 °C kPa$^{-1}$) as the tropopause is approached.

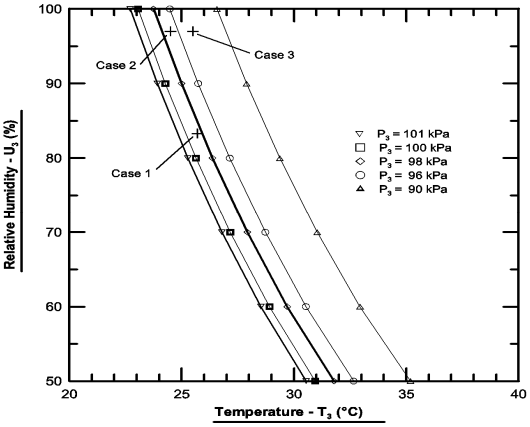

**Fig. 2 Effect of eyewall temperature and relative humidity on eyewall pressure.** Isobaric lines showing the combinations of state 3 temperature and relative humidity capable of producing a given surface pressure $P_3$. The locations of cases 1, 2 and 3 on the graph are shown with crosses. The graph is based on true-adiabatic expansion with ice ($TA_{wi}$) and $P_4 = 12$ kPa

The downward flow in the eye of a hurricane is small compared to the upward flow in the eyewall. The pressure reduction at the eye is the result of the air in the eye being entrained through friction by the eyewall. In a Rankine vortex, the pressure reduction in eye is twice the pressure reduction at the radius of maximum wind. The eyewall pressure reduction of 4.2 kPa in case 2 could therefore correspond to eye pressure reduction of 8.4 kPa; eyewall pressure reduction is approximately half of eye pressure reduction. Isabel minimum surface pressure during the IOP was 93.5 kPa, Beven and Cobb (2003).

Table 1 is based on true-adiabatic expansion with freezing of the condensed water rather than the more usual pseudo-adiabatic expansion without freezing. MPI for pseudo-adiabatic expansion without ice ($PA_{ni}$) are not much different from MPI's for true-adiabatic expansion with ice ($TA_{wi}$). For eyewall air temperature and relative humidity of 24.5 °C and 97% respectively, eyewall pressure ($P_3$) are 96.9 kPa for $TA_{wi}$ and 97.7 kPa for $PA_{ni}$. The type of expansion only has a minor effect on $P_3$ and does not significantly change the shape and position of the isobaric curves of Fig. 2. The use of true adiabatic expansion is not unique; Randall and Wang (1992,

1994) used $TA_{ni}$ and $TA_{wi}$ respectively to calculate the work produced by the raising of a 2.5 kPa layer. Freezing of the condensed water increases the work especially in TA where the condensed water does not separate from the air.

Case 1 state 1 work $W_{12}$ of 2190 J kg$^{-1}$ is identical to convective available potential energy (CAPE) for $TA_{wi}$ since all reversible processes with the same initial and final conditions must produce the same work. The use of the ideal process greatly simplifies work calculation: there is no need to calculate difference in density or virtual temperature at intermediate level. In case 1 where there is not heat addition, *the work only depends on the conditions of the air in state 1 and on the pressure and elevation at the top of the tube in state 4. The CAPE of state 3 air is zero in the three cases* because the work of convection has either been removed or dissipated upstream of state 3. The air rising in the eyewall is neutrally buoyant in accord with Emanuel (1986); indeed, the $P_3$ calculations of this paper are based on the net work in process 3-4 being zero.

At an eyewall air relative humidity ($U_3$) of 97% increasing $T_3$ from 23.3 to 23.9 °C decreases $P_3$ from 100.3 to 98.6 kPa corresponding to an increase in hurricane intensity from category 1 (33 m s$^{-1}$) to category 5 (70 m s$^{-1}$). *A 0.6 °C air temperature increase is sufficient to increase intensity from category 1 to category 5.* Actual maximum velocities would be slightly less than these ideal process velocities. The $P_3$ of 98.6 kPa for $T_3$ of 23.9 °C is almost the same as in case 1. The work of convection ($W_{12}$) of ambient air in case 1 is sufficient to produce a velocity of 66 m s$^{-1}$ corresponding to a category 5 hurricane. High ambient CAPE without heat transfer from spray is insufficient to sustain a hurricane because without spray the heat content of the boundary layer ambient air is rapidly depleted as a result of dilution by dry air. Decreasing eyewall air relative humidity by 8% has approximately the same effect on MPI as reducing eyewall air temperature by 1 °C. The eyewall pressure 96.9 kPa of case 2 could have been produced by 25.5 °C air with relative humidity of 89%. The new hurricane Isabel observations show that eyewall relative humidity is close to 97%; hurricane eyewall relative humidity could range from 92 to 98%. Holland (1997) used a relative humidity of 90% based on observed condensation level. The work ($W_{12}$) of 3750 J kg$^{-1}$ in case 2 is significantly higher (71%) than the work of 2190 J kg$^{-1}$ in case 1.

To learn a scientist must search; the right instruments must sometimes be chosen or borrowed from the tool kit of some other branch of science. The process of Fig. 1 can be analyzed using chemical engineering process simulators. These simulators have built in libraries of thermodynamic properties such as entropy and enthalpy and enforce mass and energy conservation and require rigorous process definition. Michaud (2012a) unpublished presentation shows how drag and drop simulator PROII from Simulation Sciences can be used to calculate eyewall air pressure from ambient air temperature and humidity, from SST and from spray to air mass flow ratio in a single step.

## 3. Sea-to-air heat transfer - Isenthalpic Mixing Process 2-3

There are two hurricane sea-to-air heat transfer mechanisms: *interfacial heat transfer* and *isenthalpic mixing of spray with air.* Interfacial heat transfer without spray can range from 100 W m$^{-2}$ in wind of 5 m s$^{-1}$ to 1000 W m$^{-2}$ in force 5 hurricane winds of 70 m s$^{-1}$, Ooyama (1969), Black et al. (2007), and Trenberth et al. (2007). Interfacial heat transfer has traditionally been the only heat transfer mechanism considered. It will be shown that heat transfer by spray can be 2 orders of magnitude higher than interfacial heat transfer.

Andreas and Emanuel (2001) suggested that re-entrant spray could enhance sea-to-air heat transfer and calculated the effect of spray assuming isenthalpic mixing. They realized that the spray is rapidly cooled to its wet bulb temperature. Their Fig. 1 shows that a 100 μm diameter drop of water injected in 80% relative humidity air cools to its wet bulb temperature within 1 second. *Rapid evaporation occurs as long as the vapor pressure of the liquid phase water in the drop is higher than the vapor pressure of the gaseous phase water in the air. The heat of evaporation during the initial rapid cooling phase is taken from the sensible heat of the remainder of the drop.* Once the drop has reached wet bulb temperature, evaporation is much slower because heat for further evaporation has to be transferred from the air to the drop. *Heat transfer from spray is much higher than interfacial heat transfer because drops have large surface to mass ratio.* The temperature of the water falling back in the sea approaches the wet bulb temperature of the air. Heat is transferred from the ocean to the atmosphere because the temperature of the water returned to the sea is lower than that of the water taken from the sea. *The heat transferred is essentially equal to the mass of the spray multiplied by the decrease in its specific enthalpy.*

The lower section of Table 1 shows the mass of water per unit mass of air required to increase the temperature and relative humidity of air in process 2-3. Producing air of a given specific temperature and relative humidity requires a *definite* quantity of water. In *reversible* case 2, producing air at 24.5 °C with 97 % relative humidity requires mixing either: 0.24 kg of water at 29 °C, 0.62 kg of water at 26 °C, or 1.35 kg of water at 25 °C per kilogram of air. The quantity of spray required increases towards infinity as the water temperature approaches the wet bulb temperature of the air. Table 1 shows wet bulb temperatures ($T_w$); the wet bulb of the 24.5 °C air with 97% relative humidity in case 2 state 3 is 24.1 °C. The temperature of the drops falling back in the sea would be 24.1 °C. In the 26 °C water case, evaporating 0.3% of the drop reduces its temperature to the wet bulb temperature; the water is cooled by 1.9 °C. The heat transferred per unit mass of water sprayed is 1.9 $cw$ = 8000 J kg$^{-1}$, where $cw$ = 4200 J kg$^{-1}$ is the specific heat of water. The quantity of spray required to transfer 100,000 W m$^{-2}$ is 13 kg m$^{-2}$ s$^{-1}$. In the 29 °C water case, evaporating 1.0% of the drop reduces its temperature to the wet bulb temperature; the water is cooled by 4.9 °C. The heat transferred per unit mass of water sprayed is 4.9 $cw$ = 20600 J kg$^{-1}$; the quantity of spray required to transfer 100,000 W m$^{-2}$ is reduced to 5 kg m$^{-2}$ s$^{-1}$.

Eyewall surface air temperature approaches eyewall SST irrespective of whether the work is removed from the system or dissipated within the system. *Replacing reversible isentropic expander 1-2 with an isenthalpic irreversible expander does not affect pressure $P_3$ because the rising air approaches equilibrium with the water whether process 1-2 is reversible or irreversible*. Dissipating work in an isenthalpic process reduces the heat required to produce state 3 conditions by the work dissipated. The bottom item in Table 1 shows that the water to air flow ratio is lower for irreversible expansion ($M_{7i}$ - process 1-2 constant enthalpy, $T_2 = T_1$) than for reversible expansion ($M_{7r}$ - process 1-2 constant entropy, $s_2 = s_1$). In *irreversible case 2*, producing air at 24.5 °C with 97% relative humidity requires mixing either: 0.056 kg of water at 29 °C, 0.14 kg of water at 26 °C, or 0.31 kg of water at 25 °C per kilogram of air. For the 26 °C water case, irreversibility reduces the water to air ratio of from 0.62 and 0.14, a reduction of 77%. The enthalpy supplied by the spray is reduced by the quantity of work dissipated; the heat supplied to the air is reduced from 5090 to 1340 J kg$^{-1}$, by the quantity of work dissipated in the isenthalpic process, 3750 J kg$^{-1}$. The quantity of spray required to produce a heat flux of 100,000 W m$^{-2}$ is the same in the reversible and in the irreversible cases but the air flow is 4 higher in the irreversible than in the reversible case because the majority of the heat of evaporation is provided by work dissipation. The relative humidity of any air mass can be increased to 97% with spray but the quantity of spray required is higher for lower temperature water or for dryer air. The bottom right corner of Table 1 show that increasing relative humidity to 97% requires 9 times more spray when the initial mixing ratio is 2 g kg$^{-1}$ than when the initial mixing ratio is 18 g kg$^{-1}$. Bister and Emanuel (1998) argued that dissipation can increase hurricane maximum wind by 20%; alternatively dissipation could reduce the sea-to-air heat transfer required to produce a given hurricane intensity.

In mixing process 2-3 of Fig. 1, the water and the air are mixed at once. In a hurricane the air is repeatedly sprayed with water rather than mixed with water all at once. The effect on the air of repeatedly spraying the air with small amounts of water is not much different than that of mixing all the water in the air at once. In fact spraying the water a bit at a time is slightly more effective at increasing the enthalpy of the air than adding all the water at once. In case 2 producing the state 3 air requires that each kilogram of air be sprayed with approximately 0.6 kg of 26 °C water. There is no need for the spraying to occur at once; each kilogram of air can be sprayed with 0.006 kg of water 100 times. In our view, the absence of tropical cyclones when the SST is less than 26 °C is related to the inability to produce air warm and humid enough to remain buoyant up to the upper troposphere. *The 26 °C minimum SST threshold for hurricane may be due to the fact that producing a strong hurricane requires eyewall air at a temperature of approximately 24.5 °C at a relative humidity of approximately 97% and to the fact that the quantity of spray required to produce this air is more than the spray produced if the temperature of the water is less than 2 °C higher the wet bulb temperature of the air.* Emanuel (1986) suggested that sustaining a tropical cyclone needs a transfer of heat reflected by an inward increase in relative humidity. Emanuel (1995) suggested that near saturation is a condition for intensification.

## 4. Cumulative heat transfer

A large hurricane can produce 10 mm hr$^{-1}$ of rain over a 300 km diameter area, Trenberth et al. (2007), and Ooyama (1969). Converting to rain gives a mass of 196 x 10$^{6}$ kg s$^{-1}$; multiplying by the latent heat of vaporization, the heat required to vaporize the water amounts to 491 x 10$^{12}$ W. 500 TW is an enormous amount of energy and is 250 times the world's average electrical energy production of 2 TW. Assuming that the intense heat flux takes place under the eyewall and that the eyewall has an area of 5000 km$^{2}$ gives an eyewall heat flux of 100,000 W m$^{-2}$. The average eyewall water flux is 0.04 kg m$^{-2}$ s$^{-1}$, at a mixing ratio of 20 g kg$^{-1}$ the corresponding upward air flux is 2 kg m$^{-1}$ s$^{-1}$.

A strong hurricane can cool a strip of water 100 km wide by 100 m deep by 2.5 °C and can have a speed of 5 m s$^{-1}$, D'Asaro et al. (2007), Bell and Montgomery (2008). The mass of water cooled is 50 x 10$^{9}$ kg s$^{-1}$; multiplying by the sensible heat of water and a temperature change of 2.5 °C amounts to 524 x 10$^{12}$ W. The agreement between the heat required to evaporate the rain and the heat required to cool the sea indicates that the source of the heat of evaporation is the sensible heat of the sea. The contribution of the interfacial heat flux of 1000 W m$^{-2}$ to the total sea-to-air heat flux is only 1%.

Black et al. (2007) and D'Asaro et al. (2007), show that hurricane Frances reduced water temperature by 0.5 °C along its track and by 3.2 °C in a 100 km wide band to the right of it track. The cooling takes place near core of the hurricane. It is hypothesized that the spray is mainly produced under the right eyewall where the relative wind is highest. The spray is prevented from moving inward by centrifugal force and from falling immediately back down by the upward velocity and as a result gets centrifuged out to the right of the hurricane track before falling back in the sea. The spray taken from the eyewall is replaced with warm water rising from below. Michaud (2012b) Air-Sea Interaction Conference presentation expands on this topic. Sea-to-air heat transfer in hurricane winds is extremely difficult to measure and was the subject of a recent intense study by the Coupled Boundary Air-Sea transfer Experiment (CBLAST), Black et. al. (2007). Heat transfer calculated assuming that the ocean cooling is entirely due to surface cooling could be more dependable than heat transfer measured with eddy correlation heat flux probes.

The heat capacity of the ocean is much higher than that of the atmosphere. The heat provided by cooling a layer of water 1 m thick by 1 °C is sufficient to increase the temperature of the bottom kilometer of the atmosphere by 4 °C which would be a large increase in the heat content of the atmospheric boundary layer. The heat provided by cooling a layer of water 100 m thick by 1 °C is 400 times the heat required to increase the temperature of the bottom kilometer of the atmosphere by 1 °C. Cooling a layer of water 100 m thick by 3 °C would provide a heat flux of 120,000 W $m^{-2}$ for the 3 hours duration of the eyewall passage. The heat content of the atmospheric boundary layer is not significantly different before and after the passage of a hurricane and therefore the heat must come from the sea and not from the original heat content of the atmospheric boundary layer. The heat content of the sea at temperatures above 26 °C is known as ocean heat content (OHC), Shay et al. (2000). OHC in the Caribbean can be as high as 2 GJ $m^{-2}$ enough to provide a heat flux of 120,000 W $m^{-2}$ for 5 hours. OHC plays a major role in hurricane intensity because the availability of warm water from below is essential to prevent eyewall SST from decreasing. Producing the high humidity air during the 3 hr passage of the eye could require a column of 29 °C water 43 m high, which can be a good part of the OHC. *MPI is as dependent on OHC as on eyewall SST because OHC prevents SST from decreasing*.

Emanuel (1995) investigated how the equilibrium between surface enthalpy flux and input of low entropy air from above controls the properties of the sub-cloud layer. The relative humidity of eyewall air is maintained close to 97% because the dilution by dryer air from above is balanced by evaporation of water spray from below. The water content of the air rising in the eyewall must eventually come mainly from the sea. Increasing the relative humidity of the air to 97% requires that the air be sprayed with water again and again because it is continuously diluted by dry air from above.

Hurricanes transport heat from the sea to the upper troposphere, but they do not increase the temperature of the upper troposphere in their immediate vicinity because the raised air tends to descent far from the hurricane where the subsiding air is cooler and requires less work to compress. The upper troposphere warming can take place at long distances from the hurricane and may be unnoticeable in the tropics because subsidence heating is spread over a very large area.

*Hurricane maximum potential intensity is essentially a function of eyewall SST alone* because the height of the level of neutral buoyancy and the initial relative humidity of the rising air are essentially constant. A small difference in eyewall air temperature can make a large difference in wind velocity. The TEE method is not predictive; its purpose is simply to show that wind speed and pressure reduction can be calculated from actual thermodynamic conditions.

Experience with direct contact wet cooling towers could help understand heat transfer in hurricanes because both processes are isenthalpic mixing processes. In a wet cooling tower, warm water is sprayed in air and the droplets are repeatedly broken up on splash bars. Wet direct contact cooling towers are used because of their high heat transfer rates. A cooling tower with a thermal capacity of 1000 MW might have a diameter of 100 m; the heat transfer area could be 5000 $m^2$ (an annulus with an average circumference of 300 m and a width of 17 m) resulting in a heat transfer of 200,000 W $m^{-2}$. The heat transfer per unit area in wet cooling towers is of the same order of magnitude as in the eyewall of hurricanes. Spray is a very effective heat transfer mechanism; the water at the cooling tower outlet can be 5 to 15 °C cooler than the warm water at the cooling tower inlet. Understanding the process responsible for the energy of hurricane is important because there may be a possibility of harnessing the process to produce clean and carbon free sustainable energy, Michaud and Michaud (2010) and Michaud and Renno (2011).

Emanuel (1986) calculated a hurricane heat to work conversion of 33% from Carnot efficiency. Renno (2008) estimated the overall atmospheric heat to work conversion efficiency at 20%. Converting 30% of 500 TW would result in a mechanical energy production of 150 TW. There is a potential for producing much more energy than the kinetic energy of natural wind; the kinetic energy of wind is only a small fraction of the energy that could be produced if the heat were carried upward reversibly. The energy normally dissipates since there is no mechanism in the atmosphere for capturing it.

SST and OHC both increase during the summer because ordinary non-cyclonic convection is unable to carry away as much energy as is received from net radiation even with SST as high as 30 °C, see COMET (2006) section 3.7. Increasing SST and OHC increase the probability of hurricanes developing and of becoming intense. Hurricane are far more effective than ordinary cumulus convection at carrying heat upward and once established can use heat from SST's as low as 26 °C; they play a vital role in carrying heat away from the ocean's surface.

## 5. Conclusions

Interfacial heat transfer without spray is unable to provide the heat flux required to produce either the observed precipitation or the observed sea cooling. Eyewall spray can increase sea-to-air heat transfer by a factor of 100. *Spray provides a mechanism whereby the huge heat content of the sea can quickly be transferred to the lower atmosphere.* Hurricane sea-cooling is primarily due to cooling from above and not to mixing of cold water from below. Hurricane intensity is essentially dependent on the actual eyewall SST which in turn depends on OHC. Ideal steady state thermodynamics processes and case studies could provide a new basis for understanding hurricane intensity and sea-to-air heat fluxes.

## Appendix – Calculation procedure

Pressure $P_3$ is the pressure for which the net work: the isentropic work of expansion, $h_3 - h_4$, minus the increase in the potential energy of the air, $(1 + r_3)\,gz$, in process 3-4 is zero. Hurricane eyewall air temperature and relative humidity are fairly well known from observation. The temperature and relative humidity of eyewall air were 24.5 °C and 97 % in hurricane Isabel and 26 °C and 95 % when hurricane Ophelia passed over National Moored Buoy 41049 in 2011. $P_3$ is calculated by iteration, taking case 2 of Table 1 as an example: the net work when air at 98 kPa, 24.5 °C and 97 % RH is raised is + 503 J/kg and the net work when air at 96 kPa, 24.5 °C and 97 %

RH is raised is - 416 J/kg. The pressure at which the net work is zero can then be found to be 96.91 kPa by linear interpolation. A second linear interpolation between 96.8 and 97.0 kPa gives a pressure of $P_3$ of 96.90 kPa, the lowest pressure for which the net work in process 3-4 is positive. Once $P_3$ is known, temperature $T_4$ and work $W_{12}$ can be calculated. This example is based on $P_4$ and $z_4$ of 12 kPa and 15,500 m respectively. The graph of Fig. 2 was produced using a similar interpolation approach to find the relative humidity required to produce a given pressure with air of a given temperature.

The equations used to calculate entropy and enthalpy equations are available from Michaud (2012c) where the Hewlett Packard HP48SX calculator program used to calculate both parts of Table 1 is also available. Thermodynamic properties per unit mass of dry air are commonly used because in many processes such as pseudo adiabatic expansion the mass of dry air is constant while the mass of the total substance changes. The $(1 + r_3)$ factor in the total energy equation would not be required if thermodynamic properties per unit mass of total substance were used.

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