Hurricane Isabel Intensity

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Abstract

Hurricane minimum eyewall pressure is a function of the temperature and humidity of the air at the eyewall. Hurricane maximum wind speed is a function of the difference between this eyewall surface pressure and the surface pressure at large radius. Dropsonde observations in hurricane Isabel provided unprecedented high quality data on eyewall: air temperature, relative humidity and sea surface temperature. The paper shows that eyewall temperature and humidity can be used to calculate minimum eyewall pressure and maximum wind speed.

Heavy spray at hurricane causes eyewall air temperature and relative humidity to approach equilibrium with the underlying sea surface temperature (SST). Isabel eyewall surface air temperature (SAT) was 1 to 2°C lower than its eyewall SST and its eyewall relative humidity was approximately 97%. Therefore the approach of the eyewall temperature to equilibrium (*A*) was 1 to 2°C and the approach to equilibrium of eyewall relative humidity (*B*) was 3%. Hurricane intensity is shown to be extremely sensitive to eyewall SST. An increase in eyewall SST from 26.5°C to 27.5°C is sufficient to increase hurricane intensity from category 3 to category 5. Cooling of the sea by the hurricane reduces eyewall SST to 28°C or below thereby limiting hurricane intensity. Predicting hurricane intensity is difficult because a small increase in eyewall SST has a large effect on intensity and because warm water can swell up from below.

1. Introduction

The maximum potential intensity (MPI) of hurricanes is determined by the thermodynamic properties of the ocean and atmosphere. The temperature and humidity of air rising in the eyewall are shown to be the key intensity determining factors. Several methods of establishing MPI have been proposed and the merits of the various methods have been compared in technical articles. High quality hurricane observational data for hurricane Isabel has recently become available, Montgomery et. al. 2006 (Part I hereafter) and Aberson et. al. 2006 (Part II hereafter). A maximum sustained wind speed of 76 m s^{-1} was measured near the 1000 m level (Part I) and an extreme wind maximum of 107 m s^{-1} was measured near the 1.4 km level (Part II). Isabel was at or near category 5 (surface wind speed > 67 m s^{-1}) during the observation period. Part I noted that the observed wind of 76 m s^{-1} is well in excess of the MPI of 57 m s^{-1} predicted by the Emanuel MPI (E-MPI) method. The intense observation period (IOP) took place on 13 September 2003 between 1600 and 2300 UTC when Isabel was located approximately 1300 km east of Puerto Rico.

Persing and Montgomery (2003) pointed out that Emanuel MPI (E-MPI) has gained much acceptance, but that the method can not account for modeled and observed superintensities. They suggested that a new MPI formulation from first principles is required. Holland (1997) used an iterative procedure to account for the fact that mixing ratio of air in equilibrium with water increases with decreasing pressure. Camp and Montgomery (2000), see their Fig. 4, reviewed the Holland and Emanuel MPI models;

they noted that the Holland method tends to overestimate intensity of strong hurricanes while the Emanuel method tends to underestimate the same.

The new high quality Isabel data (Part I) comprises mean sounding properties at four locations: eye, eyewall, outer core, and distant environment. Isabel data comprises sea surface temperature (SST) measurements taken at various locations before, during, and after the storm. Isabel was essentially at steady state during the IOP; eyewall surface air temperature (SAT) was 24.5°C, and eyewall relative humidity was 97%, see Part I Fig. 4.

Michaud (2001) described a method of calculating MPI based on the total energy equation, the TEE-MPI method. TEE-MPI calculates surface pressure by raising surface eyewall air approaching equilibrium with sea water at reduced eyewall pressure to its equilibrium level in a vertical tube. The TEE-MPI method is thermodynamically equivalent to the Holland iterative eyewall pressure calculation method.

This paper applies the TEE-MPI method to the Isabel data and shows that the method yields MPI in agreement with observations, see Table 1. An eyewall air temperature of 24.5°C (Part I, Fig. 4c) yields a maximum velocity of 78 m s^{-1} (Part I, Fig. 4a). An eyewall air temperature of 24°C yields a velocity of 59 m s^{-1}, and an eyewall air temperature of 25.5°C yields a velocity of 110 m s^{-1}. SST on the front side of the eyewall where full cooling has yet to take place can be 2°C higher than SST at the rear of the eyewall. The large amount of spray at the eyewall causes the eyewall air temperature to approach equilibrium with the sea. The approach of eyewall air to eyewall sea temperature in Isabel is estimated to have been 1 to 2°C. An eyewall SAT of 24.5°C would thus correspond to an average eyewall SST of 25.5 to 26.5°C.

The TEE-MPI method does not yield the extreme MPI's produced by the Holland method because eyewall SST is cooled by air-sea interaction. Cooling of the sea surface by re-entrant spray reduces eyewall SST to below 28°C. Hurricane intensity is shown to be extremely sensitive to the SST under the eyewall. Section 2 applies the TEE-MPI method to the Isabel data. Section 3 discusses the advantages of using ideal processes for analyzing the hurricane process. Section 4 discusses how hurricane wind reduces SST thereby limiting MPI.

2. Total Energy Equation MPI

The TEE-MPI method is a one dimensional steady state thermodynamics model. Fig. 1 illustrates the calculation procedure. Process 3-4 represents the upflow process in the eyewall of the hurricane. State 1 corresponds to ambient surface air well away from the hurricane. State 3 corresponds to the eyewall surface air. The air rising in process 3-4 approaches equilibrium with SST at the reduced surface pressure (P_3). State 4 is the level of neutral buoyancy of the rising air, the level at which the density of the rising air is equal to the density of the ambient air.

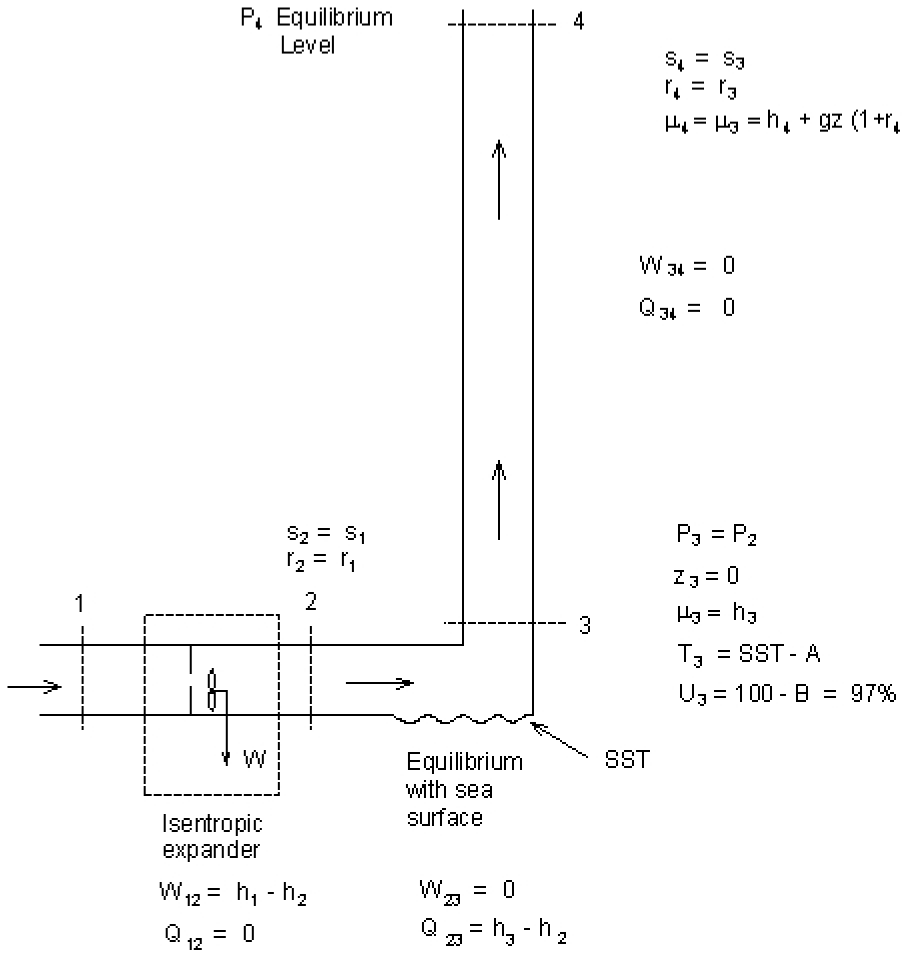

Fig. 1 Total energy equation MPI calculation concept

Entropy (*s*) is conserved during expansion processes 1-2 and 3-4. Process 2-3 represents the sea-to-air heat and mass transfer. Enthalpy (*h*) is transferred from sea to air during process 2-3; the enthalpy gain of the air is equal to the enthalpy loss of the sea. Isentropic expander 1-2 is provided to make the process ideal and thereby facilitate process analysis. The kinetic energy of the air at the outlet of the expander inlet nozzle is equal to the decrease in the enthalpy (*h*) of the air during expansion process 1-2. The kinetic energy is subsequently removed from the system by the turbine blade. The expander is used to help analyze the process and is not essential because eyewall SAT approaches eyewall SST irrespective of whether the work is removed from the system or dissipated within the system.

Table 1. Hurricane Isabel Maximum Potential Intensity (TEE-MPI). Distant surface air properties: P_1 = 101.1 kPa, T_1 = 27.8 °C, U_1 = 80, $r_1 = r_2$ = 19.06 g kg^{-1}, h_1 = 76572 J kg^{-1}, $s_1 = s_2$ = 266.8 J K^{-1}kg^{-1}. Eyewall humidity approach B = 3%, U_3 = 100 – B = 97%.

Temperature approach A = 2°C				
Eyewall SST (°C)	**26.0**	**26.5**	**27.0**	**27.5**
Temperature approach A = 1°C				
Eyewall SST (°C)	**25.0**	**25.5**	**26.0**	**26.5**
<u>Rising air properties:</u>				
$P_2 = P_3$ (kPa)	**99.14**	**97.72**	**96.01**	**94.36**
T_2 (°C)	26.12	24.90	23.41	22.72
U_2 (%)	86.8	92.2	99.3	101.75
h_2 (J kg^{-1})	74830	73557	72005	70490
T_3 (°C)	**24**	**24.5**	**25.0**	**25.5**
U_3 (%)	**97**	**97**	**97**	**97**
$r_3 = r_4$ (g kg^{-1})	18.69	19.57	20.55	21.57
$h_3 = h_4 + (1+r_4)$ gz	71686	74434	77459	80590
$s_3 = s_4$ (J K^{-1} kg^{-1})	256.2	269.7	285.2	300.8
P_4 (kPa)	15.0	10.0	10.0	10.0
T_4 (°C)	-61.45	-80.92	-77.72	-74.42
T_{4V} (°C)	-65.32	-84.69	-81.65	-78.61
T_{4A} (°C)	-62.9	-80.1	-80.1	-80.1
z_4 (m)	14220	16570	16570	16570
h_4 (J kg^{-1})	-70275	-91130	-88264	-85299
Base Pressure Reduction (kPa)				
ΔP_{12}	**1.96**	**3.38**	**5.09**	**6.74**
ΔP_2	n/a	base	1.71	3.36
Work (J kg^{-1})				
W = $h_1 - h_2$	**1742**	**3015**	**4567**	**6081**
ΔW	n/a	base	1552	3066
Velocity (m s^{-1})				
$v = (2\ W)^{0.5}$	**59.0**	**77.6**	**95.6**	**110.3**
Δv (%)	n/a	base	18.0	32.7
Sensitivity				
$\Delta P_3/\Delta T_3$ = 3.36 kPa/K				
$\Delta W/\Delta T_3$ = 3076 J kg^{-1}/K				
$\Delta v/\Delta T_3$ = 32.7 m s^{-1}/K, versus 2 m s^{-1}/K for E-MPI method.				
Heat Supplied (J kg^{-1})				
Q_{23r}	-3144	877	5454	10100
Q_{12i}	-4886	-2138	887	4018
ΔQ_{23r}	n/a	base	4577	9223
Incremental Efficiency				
n (%) = $\Delta W_{12}/\Delta Q_{23r}$	n/a	base	33.9	33.2
n (%) = $1 - T_4/T_3$	28.8	35.4	33.5	33.5

The TEE-MPI method calculates the pressure (*P_3*) at the base of a column of warm humid air rising to its neutral buoyancy level (*P_4*). TEE-MPI calculations are based on the realization that MPI is the pressure for which the work in reversible isentropic upward flow process 3-4 is zero. *P_3* is the pressure for which the net work during process 3-4, the work of expansion from *P_3* to *P_4* minus the potential energy of the raised air and water at equilibrium level *P_4* , is zero. *P_3* is calculated using a two guesses method; if a guess is too high the net work is positive; if a guess is too low the net work is negative. Interpolation in then used to determine the value of *P_3* for which the net work is zero. The TEE-MPI method is simple and direct; a single iteration is sufficient to calculate eyewall MPI (*P_3*). Once *P_3* is known expander work *W_{12}* is readily calculated by expanding ambient surface air isentropically from *P_1* to *P_2*.

The TEE-MPI method was applied to the Isabel data. Table 1 illustrates the TEE-MPI results for four eyewall air temperatures (*T_3*). Column 2 of Table 1 where *T_3* = 24.5°C matches to the Isabel data of Part I. The air at the base of Isabel's eyewall had a temperature of 24.5°C and a relative humidity of 97%, see Part 1 Fig.4. SST prior to the passage of Isabel was 27.5°C. An airborne radiometer measured a SST reduction of 1 to 2°C after the passage of Isabel, Part I. COMET (2006) microwave satellite data for Isabel track indicate that Isabel reduced SST in its track from approximately 29°C to 25°C.

Table 1 shows that an eyewall surface air temperature of 24.5°C gives an eyewall pressure of 97.7 kPa; and that an eyewall surface air temperature of 25.5°C gives an eyewall pressure of 94.4 kPa. The pressure at the eye would be somewhat lower than the pressure at the eyewall because of the forced rotation of the air in the eye. Isabel minimum eye surface pressure during the IOP was 93.5 kPa, see Beven and Cobb (2003). Isabel eye surface pressure was 3 kPa lower than Isabel eyewall surface pressure, see Bell and Montgomery (2006) Fig. 7.

P_3 depends on the initial temperature of the rising air, *T_3*, which is dependent on the eyewall SST. An updraft temperature *T_3* of 24.5°C can result from an average eyewall SST of 26.5°C and a temperature approach of 2°C, or from an average eyewall SST of 25.5°C and temperature approach of 1°C. Eyewall temperature SAT are more representative than eyewall SST because SST's are higher at the front than at the rear of the eye and because eyewall SAT tend to reflect average SST. Most Isabel STT measurements were made outside the eyewall and there were few direct SST measurements in the eyewall.

The sensitivity of eyewall pressure to SST of the TEE-MPI method is 3.36 kPa $°C^{-1}$. The sensitivity of MPI to SST of the Holland iterative method is 3.3 kPa $°C^{-1}$, see Holland (1997) Table 5. The sensitivity of E-MPI to environmental SST is 1.5 kPa $°C^{-1}$, see Holland (1997) Table 2. Eyewall pressure *P_3* is a strong function of SST and is only slightly affected by other variables. *P_3* is essentially independent of environmental surface air conditions at large radius, such as pressure *P_1*, temperature *T_1* and relative humidity *U_1*. High surface air temperature and humidity are nonetheless a prerequisite to hurricane formation.

The velocity at the outlet of the turbine inlet nozzle (*v*) is strong function of P_3 and a weak function of P_1 because pressure at large radius, P_1, is essentially constant. Correlated TEE-MPI results show that maximum velocity in meter per second and eyewall SAT are related by:

$$v = 34.3\ (\text{SAT} - 22.3) \qquad \text{for } 24°\text{C} < \text{SAT} < 26.5°\text{C} \qquad (1)$$

For a temperature approach of 2°C maximum velocity and eyewall SST are related by:

$$v = 34.3\ (\text{SST} - 24.3) \qquad \text{for } 26°\text{C} < \text{SST} < 28.5°\text{C} \qquad (2)$$

where SAT in (1) is the surface air temperature at the eyewall (T_3); SST in (2) is the average eyewall SST. While the sensitivity to eyewall SST is the same as in the Holland (1997) method, the sensitivity to pre-storm SST at large radius is much less than in the Holland method because SST under the eyewall is cooled by sea-to-air heat transfer. Average eyewall SST may never exceeds 28°C because of the cooling effect of sea spray discussed in section 4.

The sensitivity of maximum velocity to SST is much higher with the TEE-MPI method than with E-MPI method. Increasing environmental SST from 26.5°C to 27.5°C increases maximum wind from 55 to 57 m s^{-1} with the E-MPI method (2 m s^{-1} °C^{-1}), see Part 1 Fig. 7. Increasing eyewall SST from 26.5°C to 27.5°C (with a 2°C approach) increases maximum wind from 78 m s^{-1} to 110 m s^{-1} with the TEE-MPI method (32 m s^{-1} °C^{-1}), see Table 1. TEE-MPI is based on the actual eyewall SST. E-MPI is base on pre-storm SST; E-MPI must calculate eyewall SAT and eyewall SST with a sea-to-air heat transfer model.

Surface pressure, P_3, is a function of the temperature and humidity of the air at state 3 both of which are function of eyewall SST. P_3 is a strong function of eyewall SST and a weak function of the environmental sounding. P_3 is a weak function of the pressure and temperature at the equilibrium level, but the equilibrium level is correlated with SST because the higher the SST the higher the equilibrium level. Therefore (1) and (2) incorporates some of the effect of outflow temperature. P_3 is not very sensitive to the equilibrium level because the absolute value of the buoyancy is small near the equilibrium level. Raising the air to the nearest standard pressure is accurate enough for eyewall pressure calculations purpose.

Hurricanes transport heat from the sea to the upper troposphere, but they do not increase the temperature of the upper troposphere in their vicinity except in the eye and eyewall because the raised warm air tend to descent far away where the subsiding air is cool and easier to compress. The upper troposphere warming can take place at long distance from the hurricane and may be unnoticeable because it is spread over a very large subsidence area. The large quantity of heat transported upward in major hurricanes has little effect on the geo-potential height of the 10 kPa surface in the immediate area of the hurricane. Michaud unpublished manuscript: “Subsidence required to replace radiative heat loss with work of compression” discusses how subsidence in high latitudes contributes to poleward heat transport. The equilibrium level is usually close to the 10 kPa level. There is little variation in the height of the 10 kPa pressure level in maritime tropical areas. Ambient environmental sounding taken a long distance away from the center of the hurricane can therefore be used to determine the equilibrium level.

TEE-MPI calculations are based on true-adiabatic expansion with freezing of the condensed water rather than the usual pseudo-adiabatic expansion. A pseudo-adiabatic process is an isentropic process wherein the expansion occurs in stages or flashes and wherein the condensed water is separated from the air after each adiabatic expansion stage. Using pseudo-adiabatic expansion without freezing would only have a minor effect on P_3. The work is higher for true-adiabatic expansion but the extra work is used to lift water and does not significantly affect surface pressure P_3, see Michaud (1995). The equilibrium level was checked by comparing updraft temperature (T_4), updraft virtual temperature (T_{4v}), and ambient temperature (T_{4a}) for state 4, see Table 1. Environment data and elevations were taken from the mean of the San Juan sounding at the time of the Isabel observations.

3. Reversible Ideal Process

Van Ness (1969) states in his book *Understanding Thermodynamics*: "The reversible process is one for which we can readily do calculations; the alternative is likely to be that we do no calculations at all". Thermodynamic analysis requires well defined systems. The system shown in Fig. 1 is an open steady state thermodynamic system. The energy input to the open system bounded by state 1 and state 4 must equal the energy output from the system. Expander 1-2 removes work from the system because reversibility requires that work be removed rather than allowed to dissipate. The usual assumption that atmospheric expansion of rising air is isentropic must be missing something since reversible expansion requires an expander to remove work from the system.

An ideal process permits the use of standard techniques used to analyze thermodynamic flow processes. The total energy equation method has the advantage of being consistent with engineering practices for calculating flow in pipes and vessels. The velocity and kinetic energy in the tube can initially be considered to be small and therefore negligible. The pressure drop due to friction in a large diameter tube at the upward velocities encountered in hurricane is negligible therefore frictional losses can be ignored as a first approximation. Once the reversible process is understood the expander can be removed to investigate alternative ways for the work to manifest itself. Once one realizes MPI is the pressure at which the work during process 3-4 is zero processes 1-2 and 2-3 can be omitted. All that is required to calculate MPI is the temperature and relative humidity at state 3 and the elevation of the level of neutral buoyancy state 4.

Minimum hurricane eyewall pressure is the minimum pressure that can be produced by slowly raising air approaching equilibrium with eyewall SST in an insulated vertical tube extending from sea level to the equilibrium level. Maximum hurricane wind is the velocity that can be produced by expanding air from the ambient pressure at large radius to minimum eyewall pressure through an orifice or nozzle. The above two statements define the ultimate limit of hurricane intensity.

In the reversible process of Fig. 1, the work is entirely used to produce the kinetic energy of the air jet at the outlet of the turbine inlet nozzle. The kinetic energy is removed from the system when there is a turbine blade and dissipated through friction when there is no turbine blade. The ideal cycle avoids the complex problem of dissipation and associated drag factor. If not captured the kinetic energy of the jet dissipates. The ideal cycle also avoids the complex problem of sea-to-air heat transfer, the air simply approaches equilibrium with the sea. E-MPI maximum velocity depends on assumptions regarding drag coefficient and enthalpy transfer coefficients neither of which are well known.

Michaud presentation: ”Atmospheric Work Production and Dissipation” discuses how work is dissipated in unrestrained expansion.

Reversible work is a function of the inlet and outlet conditions. Therefore work is unaffected by having the tube rise at an angle provided that the inlet and outlet conditions do not changed. The eyewall can slope outwards and does not have to be vertical. The upward flow tube could have a corkscrew shape without affecting the work. The upward flow can be considered to take place in imaginary flow filaments. With the reversible expander, the maximum velocity occurs downstream of the turbine nozzle. Without the expander, the maximum velocity occurs where the flow is most restricted. Irrespective of the shape of the imaginary flow tube, the maximum velocity is limited to the maximum velocity that can be produced at the turbine nozzle.

Ideal thermodynamic processes are commonly used to facilitate the understanding of real processes. Once the ideal process where the work is removed from the system with an isentropic expander is understood it is easy to understand what can happen when the work is not removed from the system or when part of the process is irreversible. Per unit mass calculations avoid having to consider specific flows. Michaud (2000) showed that the work produced when air is raised can be calculated from the reduction in the enthalpy of a closed system provided one assumes constant entropy processes and that work is somehow removed from the system.

Reversible processes enforce consistency. Without the use of ideal process the correct SST to be used in the equations is not always clear. Camp and Montgomery (2000) noted that the Holland method is over sensitive to SST, see their Fig. 4. Tonkin et. al. (2000) noted that the Holland method simply takes the eyewall SST to be 1°C lower than the pre-storm SST. Eyewall SST can be 1 to 5°C lower than pre-storm SST. This high sensitivity attributed to Holland MPI is probably the result of basing eyewall SST on pre-storm SST minus an arbitrary amount while Holland’s derivation was based on actual eyewall SST. Tonkin et. al. (2000) noted that E-MPI assumes a constant eyewall relative humidity of between 75 to 80 %. Camp and Montgomery (2001), see their Fig. 1, noted that that MPI increases with eyewall relative humidity in the Holland method and decrease with relative humidity in the Emanuel method.

There is a real physical limitation on the eyewall SAT to SST approach. Isenthalpic mixing of 75% relative humidity air with an equal mass of water at the same temperature produces a mixture with temperature approximately 1.5°C lower than that of the starting ingredients. Doubling the water mass reduced the approach to approximately 0.75°C. The observed 1 to 2°C eyewall SAT to SST is consistent with isenthalpic mixing.

Fig. 2 extends the ideal process of Fig. 1 with more mechanical analogies. Fig. 2a is equivalent to Fig. 1 because an ideal process is unaffected by the diameter of the upflow tube. Fig. 2b is equivalent to Fig. 1 because an ideal process is unaffected by the shape of the upflow tube, thus replacing the circular upflow tube with an annular tube does not change the basic process. Fig. 2c shows that a downward flow can be produced by placing a vertical axis rotor in the base of a vertical tube; the rotor is equivalent to the impeller of a centrifugal pump. Fig. 2d shows that the function of the rotor can be replaced by the drag of the air spiraling up the outer annular tube. The rotation in Fig. 2d does not need be sufficient to pull air from the upper troposphere all the way to the surface. The downward velocity in the eye of hurricane is small compared to the upward velocity in the eyewall. The fact that eye air above the 3 km level is usually very dry indicates that it has been pulled down from the upper troposphere. The fact that eye air below the 2 km level usually has high moisture content indicates that it has lingered near the surface. This kind of reasoning shows that the downward eye flow is driven by the eyewall flow. Thus eye flow is secondary and plays no role in energy production. It is easy to show that bringing down high potential temperature air from the upper troposphere requires rather than produce energy.

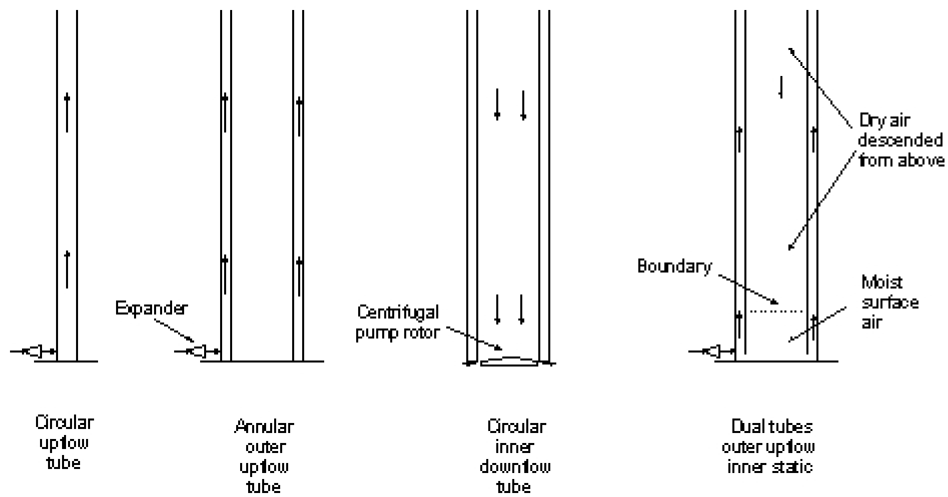

Fig. 2 Annular tube ideal processes

The TEE method readily shows that MPI is independent of the temperature T_1 and humidity U_1 of the air at large radius. Changing T_1 or U_1 has little effect on MPI because the rising air approaches equilibrium with the water at state 3 irrespective of the conditions at state 1. Changing the conditions of the air at large radius is equivalent to changing the convective available potential energy (CAPE) of the environmental sounding therefore hurricane intensity is not affected by CAPE. Persing and Montgomery (2005) used a high resolution model to show that MPI is not affected by CAPE. The TEE method shows that MPI is relatively insensitive to outflow temperature because the elevation of the 10 kPa surface is fairly constant in the tropics and because MPI is not sensitive to small changes in P_4. Michaud (2001) showed that, without sea-to-air heat transfer, the work produced in expander 1-2 becomes equivalent to a CAPE.

In Fig. 1 isentropic expansion is represented with a reversible expander wherein the work is removed from the system. Replacing the reversible expander with an irreversible isenthalpic expander has little effect on base pressure P_3 or maximum velocity because the air eventually approaches equilibrium with the SST irrespective of whether the work is removed from the system or dissipated within the system. When the work is dissipated as heat in the air, less heat transfer from the sea is required, but the final equilibrium is the same. Table 1 shows the heat transfer for the reversible expander case (Q_{23r}) and the heat transfer in the irreversible expander case (Q_{12i}). $Q_{23r} = Q_{13i} + W_{12}$. The dissipation of the kinetic energy decreases the sea-to-air heat transfer required to approach equilibrium and does not increase MPI because the eyewall SAT reaches the same equilibrium irrespective of whether the mechanical energy is removed from the system or dissipated within the system. Bister and Emanuel (1998) showed that dissipative heating can increase hurricane intensity; alternatively dissipative heating can reduce the sea-to-air heat transfer rather than increase hurricane intensity.

Gibb's phase rule states that the properties of two component system can be fully defined by specifying three properties. The Occam razor states that the number of entities should not be multiplied without necessities. Entropy is the standard thermodynamic property. The equations used to calculate the thermodynamics properties are given in Appendix 1 of Michaud (1995). Calculating entropy from potential equivalent temperature can be ambiguous because there are several ways of calculating potential equivalent temperature, Persing and Montgomery (2003). Complex models add entities in attempts to reconcile model results with observations.

Thermodynamic power cycles are invariably analyzed using closed thermodynamic systems. Attempts to understand atmospheric energy production using well defined closed thermodynamic systems have been rare.

4. Sea-to-Air mass and heat transfer

The major difference between the TEE-MPI and E-MPI methods lies in the sea to air heat transfer. TEE-MPI uses observed eyewall SAT or eyewall SST. E-MPI uses a model to calculate eyewall SST and SAT. The sea-to-air heat transfer process is complex and difficult to model, however the detail of the sea-to-air heat transfer process are not as important as the final temperature and humidity of the air at the eyewall. It does not matter whether the air receives its heat within the eyewall or outside the eyewall. The ability of the TEE-MPI method to calculate MPI from eyewall SAT has no predictive value however it confirms the validity of the zero work assumption. Isabel was essentially at a steady state during the IOP. Model results at steady state should match the observed radial profile of air and water properties. Reconciling observed and calculated steady state profiles could be a way of verifying assumption regarding sea-to-air heat transfer.

The spray under the eyewall is so heavy and the heat capacity of the water spray is so high compared to that of the air that the air rising in the eyewall approaches equilibrium with the spray. In the eyewall area, the properties of the water govern the properties of the air; the relative humidity of the air is typically around 95% and the temperature of the air is a few degrees less than the SST. The temperature approach of 1 to 2°C and the relative humidity approach of approximately 3% observed in hurricane Isabel may be common to all intense hurricanes because the spray to air heat transfer process is a common feature of all intense hurricanes.

The temperature of spray droplets tend to decrease to the wet bulb temperature of the air before the droplets fall back in the sea. The wet bulb temperature of air with a relative humidity of 80% is about 2.5°C less than its dry bulb temperature. The wet bulb temperature of air with a relative humidity of 95% is about 0.6°C less than its dry bulb temperature. Thus the temperature of the spray falling back in the sea is 2.5°C less than the temperature of the sea when the relative humidity of the air is 80% and 0.6°C when the relative humidity is 95%. Andreas and Emanuel (2001) explain how re-entrant sea-spray transfers enthalpy from sea to air.

In light wind, the heat transfer rate from the sea surface to the overlying air is in the 100-300 W m^{-2} range. In a hurricane wind the direct heat transfer from the sea surface to overlying air can be in the 500-2000 W m^{-2} range. Shay et al. (2000) estimated the ocean cooling when hurricane Opal passed over a warm eddy at 20 kW m^{-2}. The heat transfer process in a heavy spray regime is an entirely different mechanism than the normal sea-surface-to-air heat transfer mechanism. A re-entrant flow of sea-spray of 10 kg s^{-1} m^{-2} with a return temperature 0.5°C lower than SST is sufficient to account for the 20 kW m^{-2} of cooling. A re-entrant sea-spray of 10 kg s^{-1} m^{-2} is plausible and would correspond to a rain rate of 0.6 m min^{-1}. Ocean cooling comes entirely from air above whether by direct sea-to-air heat transfer or by re-entrant spray. Simulations by Chan et. al. (2001) show that a thick mixed layer attenuates SST cooling. Upflow of warm water from below transports heat upward rather than downward. A cooling rate of 20 kW m^{-2} can reduce the temperature of 1 m thick layer of water by 13°C/hr or can reduce the temperature of a 10 m thick layer by 1.3°C/hr.

The ocean cooling mechanism is not well understood. Shay et. al. (2000) estimated that 10 to 15% of the 20 kW m^{-2} (2 to 3 kW m^{-2}) is due to surface cooling and that the remainder (17 to 18 kW m^{-2}) is associated with vertical mixing. Upflow of cold water from below is not the cause of the surface cooling. The additional 17 to 18 kW m^{-2} results from re-entrant sea-spray and not from upflow of cold water from below.

Landsea et al. (2005) hurricane FAQ estimated the total upward heat flux in an average hurricane at 6 x 10^{14} W from the amount of precipitation produced by a hurricane. Assuming that half of the heat is provided by sea to air heat transfer under the eyewall, that the hurricane has an eye 60 km in diameter and that the heat transfer occurs in an annulus with an area of 5000 km^2 (an annulus with an average circumference of 250 km and width of 20 km), the eyewall heat transfer per unit area of could be 15000 W/m^2.

Engineers who design cooling towers are well aware that the warm humid air leaving the top of a cooling tower approaches equilibrium with the warm water entering the cooling tower. Intimate mixing ensures that the air approach equilibrium with the water.
The heat transfer rate in natural draft cooling towers of the type used in nuclear power plants can be over 100 kW m^{-2}. A cooling tower with a thermal capacity of 1000 MW might have a diameter of 100 m, the heat transfer area could be 5000 m^2 (an annulus with an average circumference of 300 m and a width of 17 m) giving a heat transfer of 200 kW m^{-2}. On warm days the temperature of the warm water entering a cooling tower is typically 40 to 50°C. Learning's from the cooling tower industry could be a more appropriate approach for understanding eyewall heat transfer than extending techniques used for normal wind to eyewall conditions.

E-MPI assumes that all sea-to-air heat transfer occurs during the hurricane as the air converges from a large radius. TEE-MPI recognizes that the ambient surface air may have acquired a portion of its thermal energy (enthalpy) well before the hurricane develops and that there is a boost in heat input under the eyewall. TEE-MPI recognizes that heavy spray at the eyewall can bring the air close to equilibrium with eyewall sea water temperature irrespective of the temperature and humidity at larger radius. Contributing factors to the energy boost include both the heavy spray and the fact that for a given temperature air can hold more moisture at low pressure than at higher pressure.

The heat content of the air prior to a hurricane is often sufficient to produce hurricane force wind without additional sea to air heat transfer. A CAPE of 500 J/kg is sufficient to produce hurricane velocity wind of 33 m/s. The CAPE of ambient air can be as high as 3000 J/kg corresponding to a velocity of 78 m/s. The distant surface air at the top of Table 1 has a CAPE of approximately 2700 J/kg; the pressure under a slowly rising column of ambient air would be 98 kPa. Work and heat transfer from the sea in Table 1 only increases significantly above the background CAPE once eyewall SST exceeds 25.5°C, once eyewall pressure has been reduced to approximately 97 kPa. The work is not significantly higher than CAPE until eyewall SST is 25.5°C or higher. The intense sea-to-air heat transfer under the eyewall, which requires low pressure, must play a special role in hurricane intensity.

Emanuel (2003) states that hurricanes can cool sea surface temperature by as much as 5°C. Eventually the cooled surface water sinks and the warm water from below rises in plumes or bubbles. TEE-MPI shows that a warm water bubble increasing eyewall SST from 26.5°C to 27.5°C is sufficient to increase maximum velocity from 78 to 110 m s^{-1}. A blotch of surface water 1°C higher than the mean SST can increase maximum velocity from 78 to 110 m s^{-1}. Hurricane eyewall temperatures are not uniform; eyewall temperature is warmer at the front of the hurricane than at the rear of the hurricane, see Chan et. al. (2001).

It is not uncommon for SST to have a horizontal gradient of a few degrees per 100 km. SST is gradually reduced as the eyewall moves over new water. By the time the eyewall is over an area the SST may have been reduced from 30°C to 27°C. Horizontal surface temperature gradients may not cause sudden change in hurricane intensity because the hurricane gradually cools the sea surface as it moves over new water. The cooling starts from the sea surface and gradually extends down. The top 5 to 20 meters of the sea can become cooler than the underlying water. Rapid rise of warm underlying water is more likely to cause of sudden increase in hurricane intensity than horizontal SST gradients because the warmer water can suddenly appear in the eyewall. The rise of warm water can be caused by density gradient or can be wind induced. When underlying warm water is present there is always a possibility of a rapid rise in eyewall SST and hurricane intensity. Gradual rise of warm water from below may prevent the eyewall temperature from falling to the point where hurricane intensity is reduced and not necessarily increase hurricane intensity.

Air inflow from the eye to the eyewall, see Part 1, may not be important because irrespective whether the eyewall comes from the outside or the inside of the vortex the air tends to approach equilibrium with the water spray under the eyewall. In the end the temperature of air rising in the eyewall is about 2°C less than eyewall SST temperature irrespective of its origin, and its relative humidity is around 95%.

Hurricane intensity is a self regulating process. Maximum hurricane intensity can be calculated from SST and environmental sounding alone. An eyewall SST of 30.5°C would give a minimum eyewall pressure of 85 kPa and a maximum velocity of 170 m s^{-1}, see Michaud (2001). Fortunately sea-to-air heat transfer at the eyewall increases with wind speed and thereby reduces eyewall SST and limits hurricane intensity. High wind and resulting spray tend to reduce eyewall SST to no more than 28°C thereby limiting minimum eyewall pressure to 93 kPa and maximum wind to 123 m s^{-1}. The SST's used in Michaud (2001) were higher than necessary. Table 1, 2°C approach case, shows that an eyewall SST of 27.5°C is sufficient to produce category 5 intensity. Table 1, 2°C approach case, shows that an eyewall SST of 26.5°C is required before the mixing ratio of the eyewall air, state 3, becomes higher than the mixing ratio of ambient air at large radius, State 1. More importantly, Table 1, 1°C approach case, shows that the enthalpy of the air starts to increase rapidly at eyewall SST of 26°C. It is well recognized that hurricane formation requires a minimum SST of 26°C, Holland (1997).

The passage of a hurricane over a warm eddy with a pre-storm SST of 31°C could cause the eyewall SST to increase from 26.5 to 27.5°C thereby increasing hurricane intensity form category 2 to category 5. Hurricane simulations by Chan et. al. (2001) provide valuable insight in the sea-to-air heat transfer process. The eyewall SST at the front of the hurricane can be 1 to 2°C warmer than the eyewall SST at the rear of the hurricane that has been cooled. Throwing warm water in the eyewall is like throwing oil in a fire. Without SST cooling (air-to-sea interaction), the environmental SST's in tropical seas are usually more than sufficient to produce category 5 hurricanes. Hurricane formation requires a starting impulse and therefore hurricanes are much less common than are conditions that can sustain a hurricane. Once formed hurricanes often fail to reach their maximum potential intensity for a variety of reasons including wind shear, Emanuel et. al. (2004).

Holland (1997) pointed out that direct heating from the release of all the latent heat energy of the tropical atmosphere can only reduce surface pressure form 2 to 4 kPa. He realized that heat input at reduced pressure is necessary to explain hurricane intensity. Holland estimated rather than calculated eyewall SST's. Emanuel (1986) calculated SST using a model. Neither method is capable of arriving at eyewall SST's accurate enough to calculate hurricane intensity. The Emanuel method requires an accurate model of the sea-to-air heat transfer process. The simpler Holland model would give correct MPI provided the correct eyewall SST's were available.

There are two methods of calculating the work produced in a thermodynamic engine:

1. The *efficiency* method wherein the work is the heat input multiplied by the efficiency wherein the efficiency is the Carnot efficiency calculated from hot and cold source temperatures.

2. The *enthalpy* method wherein the work is equal to the reduction in the enthalpy of the working fluid in the prime mover (expander) minus any work required to lift the working fluid.

The *efficiency* method is valuable for understanding the heat to work conversion process. The *enthalpy* method is normally used in engineering calculations because it takes into account thermodynamic properties of the working fluid including mixtures. Emanuel (1986) uses the *efficiency* method. Holland (1997) uses the *enthalpy* method. E-MPI efficiency is Carnot efficiency calculated using SST as the hot source temperature and outflow temperature as the cold source temperature. The Holland method inherently allows for heat input both prior and during the hurricane. The Emanuel method does not consider heat input that occurred prior the disturbance. The efficiencies in Table 1 were calculated using an incremental method to allow for the fact that some of the heat was received prior to the disturbance. Incremental efficiencies are defined as unit of work increase per unit of heat increase. The TEE-MPI method and the Holland iterative method calculate MPI without the explicit use of efficiency, but the efficiency is none-the-less the Carnot efficiency. The bottom two lines of Table 1 show that the incremental efficiency and the Carnot efficiency are both approximately 33 %.

5. Conclusion

Techniques used to design and troubleshoot continuous steady state processes have the potential of throwing new light on the hurricane process and should be considered. Minimum hurricane eyewall pressure is the minimum surface pressure that can be produced by slowly raising air approaching equilibrium with eyewall SST in an insulated vertical tube extending from sea level to the equilibrium level. Maximum hurricane wind speed is the velocity that can be produced by expanding air from the ambient pressure at large radius to the above minimum eyewall pressure via an orifice or flow nozzle.

MPI's calculated from eyewall SAT and relative humidity are more accurate than intensities calculated from eyewall SST because the temperature and humidity approach can vary and because dropsondes make eyewall SAT easier to measure than eyewall SST. MPI's calculated using the Holland method are less accurate than intensities calculated using actual eyewall SST because in the Holland method eyewall SST is estimated from pre-storm SST and other considerations. MPI's calculated using the Emanuel method are less accurate than intensities calculated from eyewall SST because sea-to-air heat transfer and mixing of sea layers are not understood well enough to accurately calculate eyewall SST.

An increase in eyewall SST from 26.5°C to 27.5°C is sufficient to increase hurricane intensity from category 3 to category 5. Predicting hurricane intensity will be difficult because cooling of eyewall SST and upward swell of warm water are difficult to predict. One can only be certain that a hurricane will not intensify when the hurricane has reached the maximum potential intensity for its eyewall SST and when there is no warmer water available, which is rare since SST ahead of a hurricane is usually higher than eyewall SST.

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